100 Hard Questions

Complexity: hard

1) Text 1

In 1962, Andy Warhol exhibited his artwork *Campbell's Soup Cans*, a series of 32 canvases each depicting a different variety of the company's soup cans. It has since become a tradition for many art galleries around the world to display Warhol's work. However, the artwork is trapped in its time, with outdated themes and references, so it should no longer be displayed. Art needs to create new traditions if it wants to stay relevant to contemporary audiences.

Text 2

Campbell's Soup Cans may be outdated, but it should be kept because it's a classic favorite and provides substantial income for some art galleries. Although it can be behind the times, there are creative ways to update the artwork. For example, Banksy successfully modernized the theme. His artwork *Soup Cans* combines pop art, graffiti, and other styles, and it has been gaining in popularity since it was first displayed in 2005.

Based on the texts, how would the author of Text 2 most likely respond to the underlined claim in Text 1?

A) By questioning the idea that the theme of 'Campbell's Soup Cans' is stuck in the past and by rejecting the suggestion that contemporary audiences would enjoy an updated version

B) By agreeing that contemporary audiences have largely stopped going to see 'Campbell's Soup Cans' because it's so old-fashioned

C) By pointing out that most art galleries could increase their incomes by offering modernized versions of 'Campbell's Soup Cans'

D) By suggesting that art galleries should consider offering revised versions of 'Campbell's Soup Cans' instead of completely rejecting the artwork

Complexity: hard

2) Passage 1

The surge in the application of unique nanocomposites—materials synthesized from the combination of various distinct nanomaterials, such as the fusion of iron oxide and gold nanomaterials for use in magnetic imaging—has surpassed research into the environmental hazards of nanocomposites. Regrettably, hazard assessments based on the constituents of nanocomposites are not dependable: the fusion process may modify the physiochemical properties of the constituents, resulting in harmless nanomaterials forming a nanocomposite that is far from safe.

Passage 2

The potential for increased toxicity of nanocomposites compared to the toxicity of their constituent nanomaterials has garnered warranted interest, but the effects of nanomaterial fusion differ from case to case. For example, it was recently demonstrated that a nanocomposite of silicon dioxide and zinc oxide maintained the desired optical transparency of zinc oxide nanoparticles while reducing the nanoparticles' potential to harm DNA.

Based on the passages, how would the author of Passage 2 most likely respond to the claim in the underlined section of Passage 1?

A) By agreeing that the risk mentioned in Passage 1 needs to be assessed but emphasizing that the risk is significantly outweighed by the potential advantages of nanomaterial fusion

B) By arguing that the situation outlined in Passage 1 may not be typical but admitting that the effects of nanomaterial fusion are more unpredictable than researchers initially thought

C) By denying that the scenario depicted in Passage 1 is likely to happen but recognizing that many facets of nanomaterial fusion are still not fully understood

D) By concurring that the possibility mentioned in Passage 1 is a matter of concern but noting that nanomaterial fusion does not invariably lead to that outcome

Complexity: hard

3) Passage 1

Virginia Woolf's 1928 novel Orlando stands out as an anomaly in her collection of works. Her other significant novels mainly comprise scenes from everyday life and delve deeply into their characters' inner workings, while Orlando propels itself through a series of extraordinary occurrences and examines its characters' psyche more superficially. Woolf herself occasionally viewed the novel as a lesser work, even confessing once that she 'started it as a jest.'

Passage 2

Similar to Woolf's other renowned novels, Orlando depicts how individuals' memories shape their current experiences. Like those works, it explores how individuals navigate social interactions influenced by gender and social class. Although it is more light-hearted—more entertaining, even—this literary 'jest' still seriously engages with the themes that inspired the four or five other novels by Woolf that have achieved the status of literary classics.

Based on the passages, how would the author of Passage 2 most likely react to the evaluation of Orlando presented in Passage 1?

A) By admitting that Woolf's skills were better suited to serious novels but affirming that the humor in *Orlando* is often effective

B) By agreeing that *Orlando* is less remarkable than certain other novels by Woolf but arguing that it should still be considered as a classic

C) By recognizing that *Orlando* distinctly differs from Woolf's other significant novels but asserting its importance to her collection of works nonetheless

D) By agreeing that the reputation of *Orlando* as a lesser work has led readers to neglect this novel but contending that the reputation is undeserved

Complexity: hard

4) Upon returning to Mexico in the 1930s after studying in Europe, Costa Rican-born sculptor Francisco Zúñiga, who became a naturalized Mexican citizen, underwent a dramatic shift in his artistic style. Departing from earlier academic and modernist influences, Zúñiga began sculpting

A) It expands on the preceding sentence's assertion about a pivotal point in Zúñiga's artistic journey.

B) It provides details about Zúñiga's travels to support a claim about his artistic inspirations, which is proposed in the subsequent sentence.

lifelike figures of indigenous Mexican women in a groundbreaking manner. Reminiscent of Aztec carvings and both Mexican and European folk art, these statues feature solid, simplified forms and a powerful, grounded presence.

Which choice best describes the function of the underlined sentence in the text as a whole?

C) It narrates an incident in Zúñiga's personal life that facilitated the triumph of his later career, which is summarized in the next sentence.

D) It introduces evidence that challenges the preceding sentence's depiction of Zúñiga's artistic evolution.

Complexity: hard

5) The cherry blossom tree originated in East Asia, where the beetle *Bruchidius terrenus* feeds on its seeds. In 1785, cherry blossom trees were brought to North America, a region devoid of any *B. terrenus*. However, the evolutionary ties between predators and their prey can endure over centuries and across continents. Around 2001, B. terrenus was introduced in southeastern North America near where botanist Shu-Mei Chang and her team had been observing cherry blossom trees. Within a year, 93 percent of the trees had been infested by the beetles.

Which option best explains the role of the third sentence in the overall structure of the text?

A) It posits the theory that Chang and her team aimed to investigate using cherry blossom trees and B. terrenus.

B) It introduces a broad principle that is illustrated by the discussion of the cherry blossom trees and B. terrenus.

C) It provides background that clarifies why the species mentioned migrated to new locations.

D) It proposes an alternative interpretation for the findings of Chang and her team.

Complexity: hard

6) "How authentic do they appear?" Many digital animators prioritize this query as they endeavor to construct increasingly realistic settings and illumination. Generally, while characters in digital-animated movies appear heavily exaggerated, settings and illumination are meticulously engineered to simulate reality. But some animators, such as Pixar's Sanjay Patel, are focused on a different query. Rather than initially questioning whether the settings and illumination they're constructing are convincingly authentic, Patel and others are questioning whether these elements mirror their films' distinctive narratives.

Which option best characterizes the function of the underlined query in the text as a whole?

A) It mirrors a primary objective that many digital animators have for certain components of the animations they produce.

B) It signifies a concern of digital animators who are more interested in creating unique backgrounds and lighting effects than realistic ones.

C) It expresses the uncertainty among many digital animators about how to create realistic animations using current technology.

D) It exemplifies a reaction that audiences typically have to the appearance of characters created by digital animators.

Complexity: hard

7) In the Haida language, an Indigenous language from the Pacific Northwest, 'kun-gwa' is the singular form of 'to blink,' while 'kun-ga' is the plural form; similarly, 'tlaq-tun' is the singular form of 'to tap something,' and the plural form is 'tla-tun.' These are examples of subtractive morphology, where a base word is shortened—removing the 'gwa' and 'q', in these cases—to form a new, related word. This type of subtractive morphology is prevalent in Haida.

Which choice best describes the overall structure of the text?

A) It describes the relationship between Haida and several other languages, raises a question about the nature of that relationship, and then answers that question.

B) It identifies the most frequently occurring words in Haida, explains why it is difficult to translate those words into English, and then provides examples of languages other than English into which those words can be translated.

C) It presents some specific words in Haida, describes the general linguistic phenomenon exemplified by those words, and then states that this phenomenon occurs frequently in Haida.

D) It explains the phenomenon of subtractive morphology, discusses why subtractive morphology has been controversial among scholars, and then argues that an analysis of Haida could help resolve that controversy.

Complexity: hard

8) A 2021 Nanjing Normal University study analyzed 15,000 seawater samples across 10 oceanic currents, finding particulate organic carbon-to-nitrogen (POC:PON) ratios averaged 6.8 ± 0.4 in surface waters (0–30 m) but dropped to 5.2 ± 0.3 below 30 m globally. This uniform decrease occurred despite expectations that currents like the Gulf Stream would maintain higher ratios in deeper zones.

Which choice best describes the text's structure?

A) It presents a hypothesis about current-driven nutrient variation, describes a methodology, then refutes the hypothesis with data.

B) It compares historical and contemporary views of marine stoichiometry, then reconciles them.

C) It critiques outdated sampling techniques before proposing a novel experimental design.

D) It outlines a debate about carbon sequestration, then endorses one perspective.

Complexity: hard

9) Alterations in land use and vegetation have been shown to impact carbon and nitrogen content of soil, but the extent of these effects at different soil depths remains uncertain. Yifru Abera and Taye Belachew, researchers from Debre Zeit Agricultural Research Center in Ethiopia, hypothesized that variations in land usage would result in differing carbon and nitrogen levels throughout the soil profile, not just in the surface layer (0-30 cm). They conducted a study sampling soils from various land-use types (e.g., forest, grassland, cultivated land) across multiple locations in Bale, southeastern Ethiopia. However, their findings revealed that while carbon and nitrogen levels were significantly affected by land use in the upper soil layers, these differences diminished with depth, with levels decreasing to similarly low values beyond 30 cm regardless of land-use type.

Which option best characterizes the overall structure of this passage?

A) It describes a phenomenon that scientists fully understand, explains a research team's theory about that phenomenon, and then describes findings that perfectly align with the theory.

B) It introduces an unresolved scientific question, presents researchers' hypothesis related to that question, and then describes observations that contradict their initial hypothesis.

C) It discusses a well-established scientific process, introduces competing theories about that process, and then explains how researchers concluded that all theories were equally valid.

D) It explains a widely accepted theory, discusses how researchers tested that theory, and then presents data that partially supports and partially refutes the theory.

Complexity: hard

10) Many assume that the most avid users of online shopping platforms are millennials; _____, recent studies reveal an unexpected trend towards increased usage among older generations.

Which choice completes the text with the most logical and precise word or phrase?

A) contrary to popular belief

B) in accordance with current trends

C) due to technological advancements

D) in light of demographic shifts

Complexity: hard

11) While recent studies have challenged assertions that the compositions of sixteenth-century Italian artist Michelangelo were _____ other artists of his era, it is undeniable that his residence in the heart of the Italian Renaissance in Florence meant that his works were primarily accessible hundreds of miles away from the artistic epicenter of Northern Europe.

Which choice completes the text with the most logical and precise word or phrase?

A) disregarded by

B) opposed by

C) emulated by

D) insignificant to

Complexity: hard

12) Archaeologists have found that the ancient Maya civilization used celestial bodies to mark time with remarkable _____. By observing the phases of the moon along with certain stars and constellations, the Maya were able to accurately determine the dates of important events such as equinoxes and solstices.

Which choice completes the text with the most logical and precise word or phrase?

A) inconsistency

B) precision

C) inflexibility

D) invisibility

Complexity: hard

13) While researching a topic, a student has taken the following notes:

- Researchers have invented a 'heat-cool' battery that can maintain 90% of its energy after three months.
- The battery incorporates a special type of gel that solidifies when cooled and liquefies at high temperatures.
- When the gel is in a liquid state, power circulates within the battery.
- When the gel is in a solid state, power ceases to circulate and is conserved within the battery.
- The conserved (cooled) power can be utilized by reheating (warming) the battery.

The student wants to clarify how the gel aids in energy conservation. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A) Researchers have invented a heat-cool battery that incorporates a special type of gel, which solidifies when cooled and liquefies at high temperatures.

B) The conserved energy in a heat-cool battery, which incorporates a special type of gel, can be utilized by reheating the battery.

C) When the special gel in a heat-cool battery solidifies at lower temperatures, power ceases to circulate and can be conserved within the battery.

D) Special gel allows a heat-cool battery to maintain 90% of its energy after three months.

Complexity: hard

14) While studying a subject, a learner has taken the following notes:

- Astronomers believe that the number of comets circling the Sun is in the billions.
- 81P/Wild is one of numerous comets whose orbit has altered over time.
- The path of 81P/Wild once lay between the paths of Uranus and Jupiter.
- The comet's path is now situated between the paths of Jupiter and Mars.

The learner aims to formulate and substantiate a generalization about the paths of comets. Which choice most effectively utilizes pertinent information from the notes to achieve these objectives?

A) Astronomers believe that the number of comets circling the Sun is in the billions; the comets' paths may alter over time.

B) Like Uranus, Jupiter, and Mars, billions of comets circle the Sun.

C) One instance of a comet is 81P/Wild, whose path around the Sun once lay between Uranus's and Jupiter's paths but is now situated between those of Jupiter and Mars.

D) A comet's path around the Sun may alter over time: the path of comet 81P/Wild once lay between the paths of Uranus and Jupiter but is now situated between those of Jupiter and Mars.

Complexity: hard

15) While studying a subject, a learner has recorded the following observations:

- In 1978, Sámi advocates organized demonstrations to prevent the erection of a dam on the Alta River in Norway.
- The dam would interfere with Sámi fishing and reindeer farming.
- Despite the protests, the dam was eventually constructed, but the Alta dispute had a long-lasting effect.
- It drew global focus to the matter of Sámi rights.
- It led to a series of 2005 legal safeguards affirming Sámi rights to lands, waters, and resources.

The learner aims to formulate and substantiate a general statement about the Alta dispute. Which option most effectively utilizes pertinent information from the observations to achieve this objective?

A) During the Alta dispute, Sámi advocates organized demonstrations to prevent the erection of a dam on the Alta River in Norway that would interfere with local fishing and reindeer farming.

B) Although the dam that the Sámi advocates had demonstrated against was eventually constructed, the Alta dispute had a long-lasting effect.

C) Sámi rights to lands, waters, and resources gained global focus and legal safeguards as a consequence of the Alta dispute.

D) The Alta dispute had a long-lasting effect, resulting in global focus and legal safeguards for Sámi rights to lands, waters, and resources.

Complexity: hard

16) During a study session, a learner has jotted down the following points:

- In 1971, avant-garde artist Pauline Oliveros developed Sonic Meditations.
- Sonic Meditations is not music but rather a sequence of sound-oriented tasks known as meditations.
- Each meditation comprises directives for participants to create, envision, hear, or recall sounds.
- The directives for Meditation V state, “walk so silently that the bottoms of your feet become ears.”
- Those for Meditation XVIII state, “listen to a sound until you no longer recognize it.”

The learner aims to provide a description and an instance of Oliveros’s Sonic Meditations. Which option most effectively employs pertinent information from the notes to achieve this objective?

A) Sonic Meditations is not music but rather a series of sound-based meditations that consist of instructions; Meditation XVIII, for example, instructs participants to “listen to a sound until you no longer recognize it.”

B) In 1971, Oliveros created Sonic Meditations, a series of meditations that consist of instructions for participants to make, imagine, listen to, or remember sounds.

C) “Walk so silently that the bottoms of your feet become ears” is one example of the instructions found in Oliveros’s Sonic Meditations.

D) While both meditations consist of instructions, Meditation XVIII instructs participants to “listen,” whereas Meditation V instructs participants to “walk.”

Complexity: hard

17) While conducting an investigation, a scholar has compiled the following observations:

- In North America, forests have grown into regions that were previously grasslands.
- James Thompson and H. John Peterson from Wichita State University explored whether forest growth is connected to climate alterations.
- Thompson and Peterson examined core samples from oak trees in an area that was not forested before, and correlated the trees' ages with historical weather data to check if tree populations and climate were linked.

A) James Thompson and H. John Peterson, researchers at Wichita State University, were curious if forest growth is connected to climate alterations.

B) Due to the work of James Thompson and H. John Peterson, it is now understood that dry spells may have contributed to forest growth.

- Tree population increase was linked with dry periods.
- Dry spells may have contributed to forest growth.

The scholar wishes to highlight the objective of the research project. Which option most effectively utilizes pertinent information from the observations to achieve this aim?

C) Researchers at Wichita State University have concluded that tree population increase was linked with dry periods.

D) James Thompson and H. John Peterson examined core samples from oak trees in an area that was not forested before, correlating the trees' ages with historical weather data.

Complexity: hard

18) In retrospect, considering the theories about the universe prevalent among American physicists in the 1900s, the theory of relativity was a logical progression. It might not have been a fluke, _____ that Albert Einstein and Hendrik Lorentz independently conceived the idea. Indeed, contrary to the common notion of the solitary innovator, theirs is not the first ground-breaking theory to have been developed by multiple researchers working concurrently.

Which choice completes the text with the most logical transition?

A) however,

B) then,

C) moreover,

D) for example,

Complexity: hard

19) Salvador Dali's surrealist painting *The Persistence of Memory,* a canvas featuring melting clocks amidst a barren landscape, fuses—both literally and symbolically—recognizable motifs from both realist and abstract art forms. _____ Dali's work also blurs the boundary between reality and dreams.

Which choice completes the text with the most logical transition?

A) Conversely,

B) In so doing,

C) For instance,

D) In particular,

Complexity: hard

20) In reaction to unfavorable environmental conditions, numerous plants generate *abscisic acid* (ABA), a hormone that helps them cope with stress. ABA initiates a deceleration in the biological activities of most plants. _____ when the mustard plant Schrenkiella parvula generates ABA in response to an environmental stressor, the hormone initiates rapid growth.

Which choice completes the text with the most logical transition?

A) Moreover,

B) In contrast,

C) For example,

D) Thus,

Complexity: hard

21) Socrates was of the opinion that physical entities were mere imitations of unseen ideal forms. According to him, such abstract, nonmaterial forms are the ultimate source of knowledge. On the other hand, Aristotle had a different viewpoint, suggesting that knowledge is best acquired through direct interaction with the physical world; _____ sensory experience of the physical is the ultimate source of knowledge.

Which choice completes the text with the most logical transition?

A) regardless,

B) admittedly,

C) in other words,

D) meanwhile,

Complexity: hard

22) In 1815, during his time in exile in Jamaica, the Venezuelan freedom fighter Simón Bolívar wrote a letter lauding England's republican government and expressing optimism that Latin American countries striving for liberation from Spain might accomplish something akin. The letter was directed to a local trader, Henry Cullen; _____ however, Bolívar's intention was to sway political leaders from England and Europe to back his cause.

Which choice completes the text with the most logical transition?

A) additionally,

B) ultimately,

C) accordingly,

D) consequently,

Complexity: hard

23) Economists have observed a negative relationship between the proportion of developing nations' economies that is based on agricultural production and the amount of foreign investment these nations receive. This might seem paradoxical—agriculture often requires substantial upfront investments in equipment and infrastructure that multinational companies could help provide. However, researchers explain that the cyclical nature of agriculture can contribute to currency instability and make these economies more vulnerable to external shocks, creating a level of uncertainty that foreign investors tend to avoid.

Which choice best states the main idea of the text?

A) Even though it may seem unexpected that foreign investment decreases in developing countries as agriculture becomes a larger part of their economies, this decrease happens because agriculture requires initial investments too large for foreign investors to provide.

B) Although developing countries tend to become less reliant on foreign investment as agricultural industries become a larger part of their economies, this shift may not happen if the cyclical nature of these industries destabilizes local currencies or increases countries' vulnerability to external shocks.

C) Although one might anticipate that foreign investment would increase as agriculture becomes a larger part of developing countries' economies, the reverse happens because heavy reliance on agriculture can create conditions that are unattractive to investors.

D) Although foreign investors often avoid initial investments in agricultural industries in developing countries, foreign investment may significantly increase as these industries stabilize and the associated risks decrease.

Complexity: hard

24) A 2023 World Bank study of 45 resource-dependent developing nations (e.g., Nigeria, Venezuela, DRC) found a −0.68 correlation between natural resource exports (% of GDP) and foreign direct investment (FDI). This contradicts the assumption that resource-rich nations attract more FDI, as mining/oil extraction typically requires upfront capital from multinational firms. The International Monetary Fund (IMF) attributes this paradox to the "resource curse": price volatility in commodities like oil (which swung between $23 and $131/barrel from 2015–2023) destabilizes currencies, fuels inflation (e.g., Angola's 27.3% average annual inflation 2016–2022), and

A) Foreign investment decreases in resource-dependent nations because environmental regulations in mining sectors create compliance costs that outweigh profit potential.

B) Developing nations with large natural resource sectors attract less FDI due to investor concerns about political corruption, with 78% of extractive-industry FDI diverted to offshore accounts (UNCTAD 2022).

increases sovereign default risks (12 of 15 Sub-Saharan African resource exporters faced debt distress by 2023). These systemic risks deter long-term FDI despite short-term extraction opportunities.

Which option best summarizes the empirical relationship described in the passage?

C) A negative correlation exists between natural resource dependence and FDI in developing economies because commodity price volatility generates macroeconomic instability that discourages sustained foreign investment.

D) Foreign investment increases proportionally with natural resource exports until nations reach middle-income status, after which diversification reduces reliance on extractive sectors.

Complexity: hard

25) For many years, the general acceptance of the role of the cerebellum—a complex brain structure in vertebrates—in managing motor control in humans has overshadowed the potential for other roles of this structure. Studies in neuroscience over the past twenty years now propose that the cerebellum controls emotion and social behavior, and recent research by Ilaria Carta and her team has discovered a link between the cerebellum and a hub for motivation and reward processing known as the ventral tegmental area (VTA).

Which option best summarizes the central concept of the text?

A) The recent confirmation of a link between the VTA and the cerebellum validates the cerebellum's long-presumed role in motor coordination.

B) Recent breakthroughs in neuroscience have questioned universally accepted beliefs about the function of a link connecting the VTA and the cerebellum.

C) The cerebellum has primarily been believed to control motor functioning, but in recent years neuroscience researchers have been revealing additional roles.

D) Historical technological constraints have impeded the study of the cerebellum, but the recent advent of new technologies has led to deeper insights into its roles.

Complexity: hard

26) After penning the fervent plea 'Letter to the Spanish Americans' in 1791, Peruvian scholar Juan Pablo Viscardo y Guzmán is frequently viewed as a precursor to the independence movements in Latin America. However, without Venezuelan revolutionary Francisco de Miranda, who received the unpublished letter following Viscardo's demise, Viscardo's historical significance would have been negligible. Miranda not only disseminated the letter, but his revisions and annotations to the text place him as a key player in the formation of the text.

Which option accurately conveys the primary concept of the text?

A) The original authorship of 'Letter to the Spanish Americans' is questioned by modern historians.

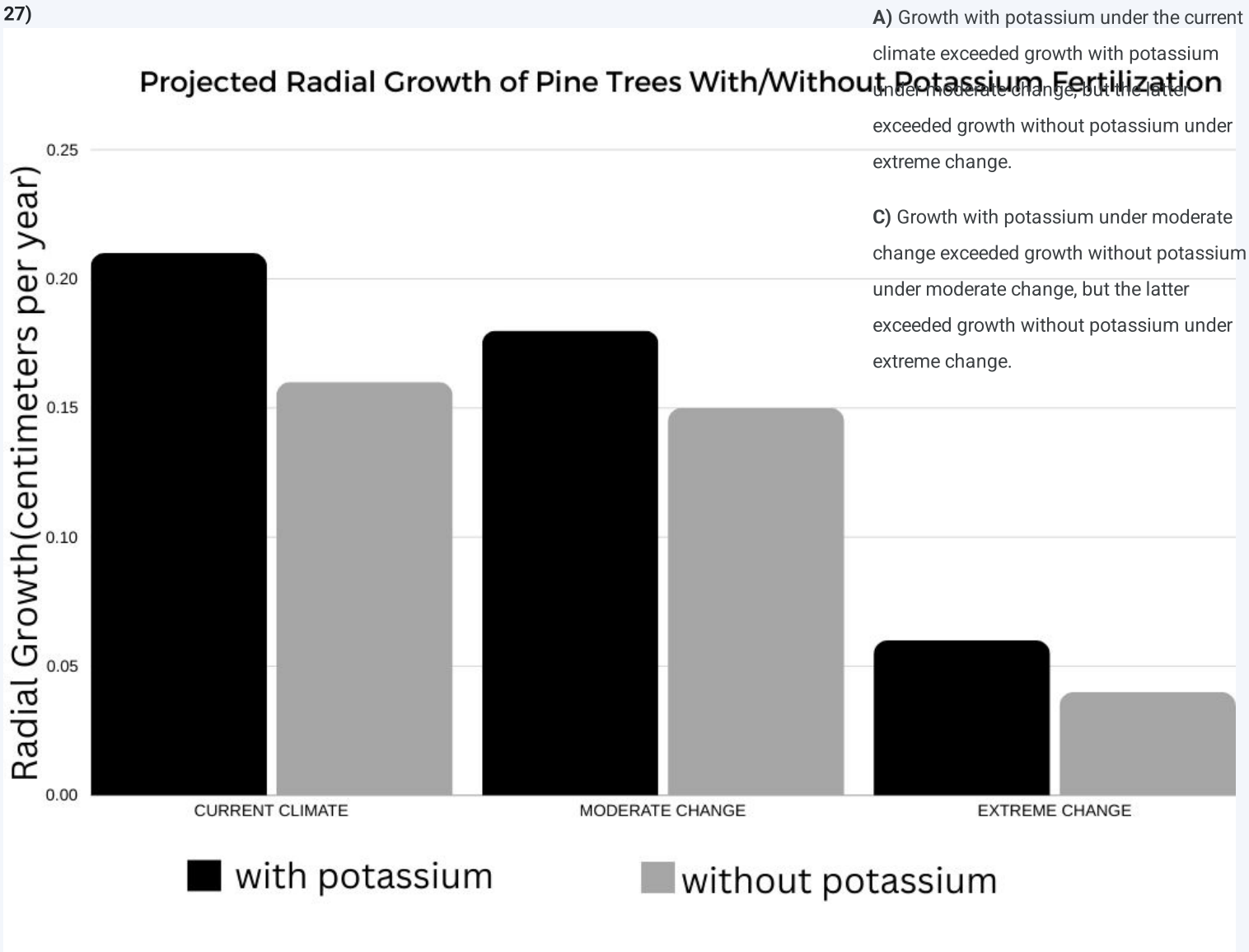
B) The bulk of the most articulately framed arguments in 'Letter to the Spanish Americans' were penned by Miranda.

C) Miranda had a pivotal role in shaping the content and dissemination of 'Letter to the Spanish Americans.'

D) 'Letter to the Spanish Americans' inspired numerous individuals in Latin America to seek national independence.

Complexity: hard

27)



A) Growth with potassium under the current climate exceeded growth with potassium under moderate change, but the latter exceeded growth without potassium under extreme change.

B) Growth without potassium under the current climate exceeded growth without potassium under moderate change, but the latter exceeded growth with potassium under extreme change.

C) Growth with potassium under moderate change exceeded growth without potassium under moderate change, but the latter exceeded growth without potassium under extreme change.

D) Growth with potassium under moderate change exceeded growth without potassium under the current climate, but the latter exceeded growth with potassium under extreme change.

Dr. James Thompson and his team performed a study on a forest site where some pine trees were periodically fertilized with potassium to mimic the global trend of increasing anthropogenic potassium deposition in soil. Thompson and his team modeled the radial growth of the trees with and without potassium fertilization under three different climate scenarios (the current climate, moderate change, and extreme change). They found that climate change would have a negative impact on growth, but they concluded that anthropogenic potassium deposition could potentially offset that effect if the change is moderate rather than extreme.

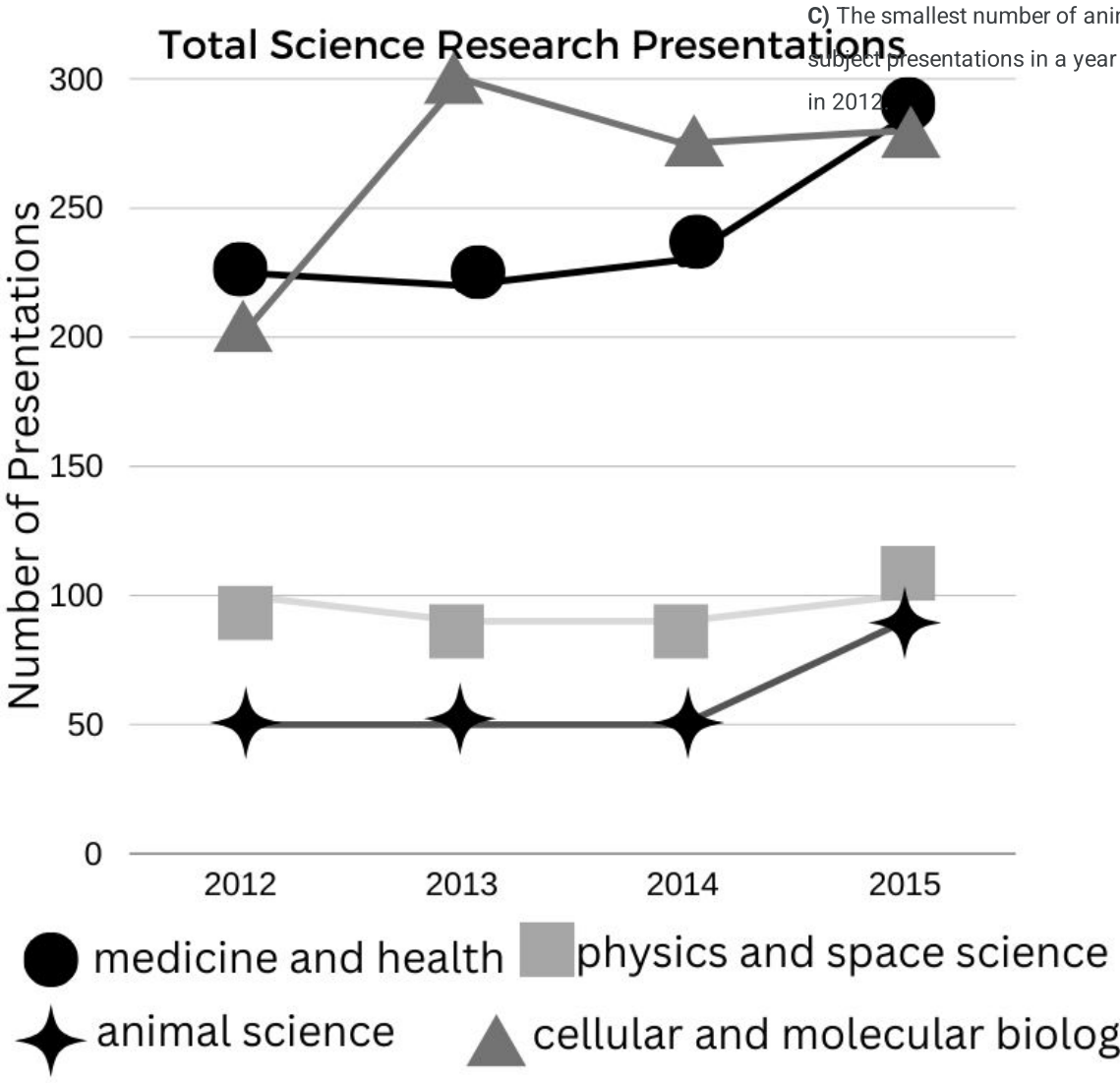
Which choice best describes data from the graph that support Thompson and his team's conclusion?

Complexity: hard

28)

A) In 2012, the quantity of cellular and molecular biology subject presentations was identical to the quantity of animal science subject presentations.

B) In 2015, there were more physics and space science subject presentations than there were medicine and health subject presentations.



C) The smallest number of animal science subject presentations in a year was roughly 95 in 2012.

D) The largest number of medicine and health subject presentations during the period shown is approximately 285 in 2015.

A scholar is examining the patterns in the subjects presented to a nationwide scientific competition for secondary school learners. The chart illustrates the quantity of presentations by subject that were made annually. Based on the information in the chart, the scholar asserts that there were a greater number of medicine and health research subjects presented in 2015 than in any other year.

Which option most effectively employs data from the chart to back up the underlined assertion?

Complexity: hard

29) Concentrations of Nucleobase from Murchison Meteorite and Soil Samples Measured in Parts per Billion

Nucleobase	Sample of Murchison meteorite 1	Sample of Murchison meteorite 2	Soil sample from Murchison
Isoguanine	0.5	0.04	not detected
Purine	0.2	0.02	not detected
Xanthine	39	3	1
Adenine	15	1	40
Hypoxanthine	24	1	2

Using high-performance liquid chromatography—a method that employs pressurized water to segregate material into its constituent molecules—astrochemist Yashiro Oba and his team examined two samples of the Murchison meteorite that fell in Australia and soil from the meteorite's landing site to ascertain the concentrations of various organic molecules. By contrasting the relative concentrations of molecules known as nucleobases in the Murchison meteorite, with those in the soil, the team inferred that the nucleobases in the Murchison meteorite originated in space and are not the result of contamination on Earth.

Which choice best describes data from the table that support the team's conclusion?

A) Isoguanine and purine were detected in both meteorite samples but not in the soil sample.

B) Adenine and xanthine were detected in both of the meteorite samples and in the soil sample.

C) Hypoxanthine and purine were detected in both the Murchison meteorite sample 2 and in the soil sample.

D) Isoguanine and hypoxanthine were detected in the Murchison meteorite sample 1 but not in sample 2.

Complexity: hard

30) Proportion of Accessible Eggs Consumed by Cane Toad Tadpoles

Amphibian species (common name)	Proportion of eggs consumed	Native to Australia	Produces bufadienolide
Tiny crimson tree frog	1%	yes	no
Cane toad	90%	no	yes

A) The tadpoles consumed a higher percentage of the striped digging frog eggs than they did of the eggs of the delicate green tree frog.

B) The tadpoles left a certain percentage of the eggs of each of the five species untouched, thus ultimately allowing them to hatch.

C) The tadpoles consumed a lower percentage of the short-legged frog eggs than they did of the eggs of their own species.

D) The tadpoles consumed the same percentage of the delicate green tree frog eggs as they did of the tiny crimson tree frog eggs.

Short-legged frog	7%	yes	no
Striped digging frog	10%	yes	no
Delicate green tree frog	1%	yes	no

Originally from Latin America, the cane toad was brought to Australia in the 1930s. In the past few decades, it has been observed that Australian tadpoles consume eggs of their own species. A 2022 study indicated that when given a choice between cane toad eggs and eggs of native Australian amphibians, cane toad tadpoles predominantly consumed eggs of their own species. This behavior stems from their attraction to bufadienolide, a compound produced by the eggs of cane toads but not by the eggs of native amphibians. However, a student, using the data from this study, argues that the presence of bufadienolide doesn't fully account for the cane toad tadpoles' preference for certain eggs over others.

Which choice best describes data from the table that support the student's argument?

Complexity: hard

31) In the natural state at the Earth's surface, water molecules form a tetrahedral network stabilized by hydrogen bonds between neighboring molecules. However, extreme high pressure, such as that found in the deep ocean, destabilizes these bonds and compresses the structure of water, allowing water molecules within organisms to permeate proteins and disrupt essential biological functions. Nevertheless, deep-sea organisms known as piezophiles have adapted to these extreme pressures. Research has shown a positive correlation between the depths at which various piezophiles live and the concentrations of a compound known as trimethylamine N-oxide (TMAO) in their muscle tissues. This has led a group of researchers to hypothesize that TMAO reduces the compressibility of water.

Which discovery, if proven true, would most directly support the researchers' hypothesis?

A) It is discovered that water molecules are resistant to TMAO even when the tetrahedral configuration of the water molecules has been distorted by high pressure.

B) A study of TMAO's molecular structure reveals that TMAO molecules maintain their form even as pressure increases.

C) A positive correlation is discovered between concentrations of TMAO and the rate at which the molecular structure of water compresses as pressure increases.

D) An analysis of the molecular structure of water under high pressure reveals that hydrogen bonds are more stable when TMAO is present than when it is not.

Complexity: hard

32) The impact of store lighting on consumer behavior has been the subject of numerous studies. Retailers often manipulate lighting conditions to create a specific mood or atmosphere, encouraging customers to stay longer and make more purchases. In a recent study, a sociologist and his team discovered that a similar effect can be achieved by varying the store's lighting conditions. They found that when customers are exposed to changing light patterns, they tend to explore more sections of the store, increasing their exposure to a broader range of products and thus boosting the likelihood of impromptu purchases.

Which response from a survey given to shoppers who made a purchase at a retail store best supports the researchers' explanation?

A) I went to buy some cosmetics, but the store had a unique lighting pattern. While I was observing it, I noticed a new skincare product and decided to buy it.

B) I didn't buy everything on my shopping list today. Some of the items were not available in the store, even though I searched everywhere.

C) I received a promotional email for a new brand of shampoo, so I visited the store to check it out. I ended up buying a few bottles because of the promotion.

D) This store has a larger product range than the one near my place. I came here specifically to buy some items that are not available at the other store.

Complexity: hard

33) Language expert Laura Thompson has warned against presenting contentious topics in terms of two extremely competitive viewpoints, such as for and against. Thompson argues that this debate-oriented method can oversimplify issues and, when employed in front of an audience, can be less enlightening than the delivery of multiple viewpoints in a non-antagonistic format. To validate Thompson's theory, students executed a study where they presented participants one of three different versions of local news commentary about the same topic. Each version included a debate between two commentators with contrasting opinions, a panel of three commentators with diverse opinions, or a single commentator.Which result from the students' study, if accurate, would most powerfully validate Thompson's theory?

A) On average, participants perceived commentators in the debate as more informed about the topic than commentators in the panel.

B) On average, participants perceived commentators in the panel as more informed about the topic than the single commentator.

C) On average, participants who viewed the panel correctly answered more questions about the topic than those who viewed the debate or the single commentator did.

D) On average, participants who viewed the single commentator correctly answered more questions about the topic than those who viewed the debate did.

Complexity: hard

34) In the 1990s, numerous artists and critics in the English-speaking world started to highlight music from different parts of the world—like flamenco from Spain and koto music from Japan—that doesn't fit neatly into British or North American popular music genres, often labeling such music as 'world music.' While some academics have appreciated this trend for bringing varied musical styles to the forefront in countries where they had previously been ignored, musicologist Alex Johnson argues that the notion of world music homogenizes extremely diverse traditions by bundling them all into a single category.

Which discovery about flamenco and koto music, if proven true, would most strongly validate Johnson's argument?

A) Flamenco and koto music evolved independently of each other and share few musical similarities.

B) Flamenco is significantly more favored in the English-speaking world than koto music.

C) Flamenco and koto music are now performed by a diverse range of artists with no direct links to Spain or Japan.

D) Flamenco and koto music are highly different from British and North American popular music genres but similar to each other.

Complexity: hard

35) In a 2018 study, Dr. Emily Peterson, Dr. John Davis, and Dr. Michael Thompson compared the foraging behaviors of wild rabbits in Australia's Flinders Ranges National Park with those of captive rabbits in England's London Zoo. The researchers observed that previous studies on various animals have indicated that providing animals with food at regular intervals, as is typical in captive settings, may unintentionally foster the emergence of new stereotypic (i.e.,

A) captive rabbits showed a stereotypic behavior of repeatedly digging in the same spot, whereas wild rabbits did not.

B) during feeding times, captive male rabbits showed much more aggression than did wild male rabbits, whereas female captive and wild rabbits showed similar levels of aggression.

purposelessly repetitive) behaviors by decreasing the need for a high level of cognitive engagement with the environment; the researchers were therefore not entirely surprised to find that _____

Which choice most logically completes the text?

C) when caretakers placed food in boxes that were cognitively demanding to open, captive rabbits showed repeated behaviors similar to those that wild rabbits show when foraging.

D) captive rabbits showed a stereotypic behavior of hopping in their enclosures as feeding times approached, whereas wild rabbits showed a stereotypic behavior of hopping before embarking on a forage.

Complexity: hard

36) In a recent study published in Scientific Reports, researchers from Eötvös Loránd University in Hungary investigated the sleep patterns of hand-raised, socialized wolves using non-invasive EEG methods. The study aimed to compare the sleep stages of wolves with those of dogs to better understand the effects of domestication on sleep physiology. Researchers found that while wolves and dogs share similarities in their sleep stages, wolves tend to spend more time in REM sleep, which is associated with neurodevelopment, stress, and memory consolidation. The findings also highlighted that older wolves exhibit more pronounced differences in REM sleep compared to dogs. The researchers were therefore not surprised to find that _____.

Which choice most logically completes the text?

A) Wolves showed a higher proportion of REM sleep compared to dogs, particularly in older individuals.

B) Dogs displayed more aggressive behaviors during sleep disturbances than wolves, regardless of age.

C) Wolves exhibited stereotypic pacing behaviors before entering REM sleep, unlike dogs.

D) Wolves and dogs have identical sleep patterns across all stages of sleep.

Complexity: hard

37) In 2020, ornithologist Dr. Peter Marra and his team explored the evolution of the syrinx, a vocal organ present in birds, but is generally thought to have no function in other animals. Studying 350 bird species, the team found that the syrinx has developed independently across multiple lineages in separate instances and, significantly, hasn't disappeared after developing in specific lineages. Moreover, the team determined that species with the organ tend to have more complex vocalizations. Therefore, the team hypothesized that the syrinx likely _____

Which choice most logically completes the text?

A) was once present in many non-avian species but has since disappeared from those lineages.

B) has been preserved in certain bird species because it benefits their communication systems.

C) will develop in a greater number of bird species because it may serve a necessary function in the communication system.

D) produced more complex vocalizations in birds in the past than it does currently.

Complexity: hard

38) In 2018, marine biologist Dr. Laura Robinson led an international team on a deep-sea expedition aboard the research vessel JOIDES Resolution to study cold-water corals in the Atlantic Ocean. By analyzing the chemical composition of coral skeletons, the team reconstructed past ocean temperatures and oxygen levels. These cold-water corals, unlike their tropical counterparts, thrive in the dark, nutrient-rich depths of the ocean. The team noted that such corals often coexist with fish species exhibiting highly efficient gill structures and elevated levels of oxygen-carrying hemoglobin, adaptations likely influenced by the low-oxygen environments in which they live. Based on these observations, the researchers hypothesized that gill specialization in these species likely _____.

Which choice most logically completes the text?

A) originated in land-dwelling vertebrates before being adapted to aquatic life.

B) evolved primarily for temperature regulation rather than oxygen intake.

C) contributes to survival in low-oxygen deep-sea habitats.

D) leads to rapid increases in hemoglobin concentration during coral spawning.

Complexity: hard

39) The Inca Empire, one of the most sophisticated pre-Columbian civilizations, emphasized reciprocity (*ayni*) and community well-being as central principles in its societal organization. The Incas believed in a system where individual actions were evaluated based on their contribution to the collective good and their alignment with societal roles. This contrasts with ethical systems that judge actions based solely on universal principles, independent of context. Assuming this understanding is accurate, it can be inferred that _____.

Which choice most logically completes the text?

A) the Inca ethical framework suggests that the morality of an individual's actions was judged primarily by their alignment with the individual's societal role and their contribution to communal harmony.

B) There is no evidence suggesting that the Incas relied on external observers to determine morality; their ethical system was deeply rooted in their own societal structure.

C) This contradicts the Inca approach, which prioritized context-dependent evaluations tied to societal roles and contributions.

D) The Incas did not view all actions as morally equivalent; they judged them based on their impact on community outcomes and alignment with societal roles.

Complexity: hard

40) On April 15, 1912, a tragic event shook the world. The *Titanic*, a luxury passenger liner, struck an iceberg in the North Atlantic Ocean—a major shipping route between Europe and North America. The ship took less than three hours to _____ despite being touted as unsinkable.

Which choice completes the text so that it conforms to the conventions of Standard English?

A) sink, largely due to its size and the damage sustained,

B) sink; largely due to its size and the damage sustained:

C) sink, largely due to its size and the damage sustained:

D) sink; largely due to its size and the damage sustained,

Complexity: hard

41) The epic poem *Beowulf* commences with the term 'hwæt,' an Old English _____ to 'hark!' or 'listen!' in certain versions, amusingly interpreted as 'dude!' by Maria Dahvana Headley in her 2020 rendition of the poem.

A) exclamation, translated

B) exclamation and translated

C) exclamation translated

D) exclamation. Translated

Which option best completes the following sentence so that it conforms to Standard English?

Complexity: hard

42) The acclaimed body of work by Nigerian author Buchi Emecheta includes *The Joys of Motherhood*, a novel that explores the evolving roles of women in 1950s _____ a television drama that delves into the personal challenges faced by a newly married couple in Nigeria; and *Head*

A) Lagos, *A Kind of Marriage*,

B) Lagos; *A Kind of Marriage*,

C) Lagos, *A Kind of Marriage*:

D) Lagos; *A Kind of Marriage*

Above Water, her autobiography.

Which choice completes the text so that it conforms to the conventions of Standard English?

Complexity: hard

43) In 1994, nearly two centuries post the demise of Wang Zhenyi, the Global Astronomical _____ the achievements of the pioneering 18th-century astronomer and author of *Dispute of the Procession of the Equinoxes*, by naming a crater on Venus in her honor.

A) Union would finally recognize

B) Union to finally recognize

C) Union, having finally recognized

D) Union, finally recognizing

Which choice completes the text so that it conforms to the conventions of Standard English?

Complexity: hard

44) In 1986, Homero Aridjis, a prominent Mexican environmental activist and founder of the Group of 100, played a pivotal role in securing government protection for the forests where monarch butterflies overwinter. This achievement marked a significant milestone in Mexico's environmental conservation efforts. His advocacy _____ instrumental in ensuring the preservation of these critical habitats.

A) are

B) was

C) have been

D) were

Which option completes the passage so that it adheres to the norms of Standard English?

Complexity: hard

45) A second-generation Japanese American, Wataru Misaka _____ in World War II (1941–45) and won two amateur national basketball championships at the University of Utah before he joined the New York Knicks for the 1947–48 season, becoming the first non-white basketball player in the Basketball Association of America (BAA), which later became the NBA.

A) had already served

B) already serves

C) was already serving

D) already served

Which option completes the text in a way that it aligns with the norms of Standard English?

Complexity: hard

46) French philosopher René Descartes revolutionized Western philosophy with his famous dictum, *"Cogito, ergo sum"* (I think, therefore I am). This principle asserts that the act of thinking is proof of existence. The _____ complexity: only those who think can affirm their existence.

A) quotes' simplicity masks its

B) quotes simplicity masks their

C) quote's simplicity masks their

D) quote's simplicity masks its

Which choice completes the text so that it conforms to the conventions of Standard English?

Complexity: hard

47) The literary prowess of John Steinbeck, most recognized for his 1939 novel *The Grapes of Wrath* and the discourse it generated, _____ confined to the domain of fiction: he also wrote journalistic pieces, especially later in his career, where his enduring focus on the struggles of ordinary people—something he viewed as a timeless theme—further shaped his literary identity.

A) haven't been

B) aren't

C) isn't

D) wasn't

Which choice completes the text so that it conforms to the conventions of Standard English?

Complexity: hard

48) When first observed in the dense forests of Madagascar in 1863, the lemur-like aye-aye puzzled scientists and was classified as Daubentonia madagascariensis (sometimes referred to as "Madagascar's Daubentonia"). Although it is a nocturnal feeder, this creature occasionally basks in _____ sunlight during the day.

A) they're

B) their

C) its

D) it's

Which choice completes the text so that it conforms to the conventions of Standard English?

Complexity: hard

49) James Patterson is a prolific author known for his broad readership and seemingly effortless writing style. While his narratives may appear to flow naturally, Patterson attributes this ease to extensive preparation. He carefully outlines each story in advance and treats every project as if he is encountering the material for the first time. Despite his experience, Patterson does not rely solely on instinct or routine; rather, he approaches each novel with the same level of scrutiny and planning as he would a new challenge.

A) To provide details about how Patterson chooses the genre for his novels.

B) To assert that Patterson's writings appear effortless because of his preparation.

C) To discuss the types of stories Patterson feels most comfortable encountering for the first time.

D) To describe the unique way that Patterson approaches stories he hasn't written before.

Which choice best states the main purpose of the text?

Complexity: hard

50) Shifts in the composition of wildlife due to habitat destruction and other human activities can affect the levels of carbon and nitrogen in the ecosystem, though the depth of these impacts is not fully known. Proposing that variations in habitat types lead to differences in carbon and nitrogen levels that are not limited to the surface layer (0-30 cm deep), Dr. Samuel Harper and his team sampled soils across multiple habitat types (e.g., grassland, woodland, wetland) within each of several North American locations. They observed, however, that across these habitat types, carbon and nitrogen decreased to comparably low levels beyond depths of 30 cm.

Which choice best describes the overall structure of the text?

A) It describes a phenomenon that scientists do not fully understand, explains a research team's hypothesis about that phenomenon, and then describes a finding that led the team to refine the hypothesis.

B) It introduces an unresolved scientific question, presents a research team's hypothesis pertaining to that question, and then describes an observation made by the team that conflicts with that hypothesis.

C) It discusses a process that scientists are somewhat unclear about, introduces competing hypotheses about that process, and

D) It explains a hypothesis that has been the subject of scientific debate, discusses how a research team tested that hypothesis, and then

then explains how a research team concluded that one of those hypotheses is likely correct.

presents data the team collected that validate the hypothesis.

Complexity: extraHard

51) Let $k = a + b + c$, where a, b, and c are real numbers.

Consider the expression $x^2 - a^2 - b^2 - c^2 - 2ab - 2ac - 2bc + 2x(a + b + c)$.

Which of the following expressions is equivalent to the given expression in terms of x, k, and the sum $ab + bc + ac$?

A) $x^2 + 2xk - k^2$

B) $x^2 + 2kx - k^2 + abc$

C) $(x + k)^2$

D) $(x + a + b)^2 + c^2$

Complexity: extraHard

52) If $r = \sqrt[5]{k}$, and $r^{15a-8} = k^3$, what is the value of a?

A) $\frac{11}{15}$

B) $\frac{23}{15}$

C) $\frac{13}{15}$

D) $\frac{7}{5}$

Complexity: extraHard

53) The function $f(x) = a^x - b$ passes through the points (p, q) and $(2p, 4q)$, where $a^p = 2q$. What is the value of b?

A) $\frac{5}{9}$

B) $\frac{15}{84}$

C) $\frac{5}{4}$

D) $\frac{9}{4}$

Complexity: extraHard

54) The function $f(x) = A \cdot b^{\sqrt{x}}$, where A and b are positive constants and $b > 1$, models the growth of a bacterial culture. If $f(x)$ increases by exactly $c\%$ when x increases from 9 to 16, express c in terms of b.

A) $c = 100(b^4 - b^3)$

B) $c = 100(b^{4-3} - 1)$

C) $c = 100(b^{\sqrt{16}} - b^{\sqrt{9}})$

D) $c = 100 \cdot \log_b\left(\frac{f(16)}{f(9)}\right)$

Complexity: extraHard

55) The function $f(x) = ab^x$, where a and b are positive constants, models the decay of a radioactive substance in grams over time in hours.

A laboratory technician records that after $m - 2$ hours, the mass satisfies the equation:

$f(m - 2) = 1.2f(m) + 0.64f(m - 1)$

What is the value of b?

A) 0.2096

B) 0.329

C) 0.684

D) 0.508

Complexity: extraHard

56) If $\left(\frac{2}{5}x - \frac{37.5}{5}\right)^2 - 100 = 0$, what is the value of $(x - 18.75)^6$?

A) 5^{12}

B) 244140635

C) 5^7

D) 5^{10}

Complexity: extraHard

57) The exponential function $f(x)$ satisfies $f(3) = 640,000$, and the equation is of the form $f(x) = A \cdot B^{x+C}$, where A, B, and C are constants. Which of the following equivalent forms of the function f shows the value of $k = f(4)$ as the coefficient or the base?

A) $f(x) = 2,500 \cdot (5)^{x+1}, \quad k = 1,562,500$

B) $f(x) = 2,500 \cdot (5)^{x-1}, \quad k = 62,500$

C) $f(x) = 2,500 \cdot (4)^{x+2}, \quad k = 2,560,000$

D) $f(x) = 2,500 \cdot (4)^{x+1}, \quad k = 2,560,000$

Complexity: extraHard

58) The value of a rare sculpture is modeled by the function $V(t) = 2000(1.10)^{\frac{t-2005}{3}}$

where $V(t)$ is the value in dollars and t is the year.

According to this model, the value of the sculpture increases by a fixed percentage every n years.

What is the value of n?

A) 1

B) 5

C) 3

D) $\frac{-2005}{3}$

Complexity: extraHard

59) If $g(x + 2) = \frac{1}{9}g(x)$, which of the following could represent the function $g(x)$?

A) $g(x) = 9\left(\frac{1}{9}\right)^x$

B) $g(x) = 27\left(\frac{1}{3}\right)^x$

C) $g(x) = \frac{1}{3}(3)^x$

D) $g(x) = \left(\frac{1}{9}\right)^x$

Complexity: extraHard

60) If $\sqrt{579 + 48\sqrt{3}} = r + \sqrt{p}$ where r and p are positive integers, what is the value of $r + p$?

A) 55

B) 61

C) 27

D) 31

Complexity: extraHard

61) $(x + 2)^2 + (x^2 + 4x) = (x + 2)(x - 3)$

In the given equation, $x \neq -2$. What is the solution to the given equation?

A) $x = \frac{-9 \pm \sqrt{41}}{2}$

B) $x = \frac{-9 \times \sqrt{41}}{2}$

C) $x = \frac{-8 \pm \sqrt{41}}{2}$

D) $x = -\frac{9 \pm \sqrt{43}}{2}$

Complexity: extraHard

62) In the xy-plane, a line representing the revenue in thousands of dollars from a certain business is given by the equation $y = -1.5x + m$, where m is a constant. The company's projected cost, in thousands, follows a quadratic trend modeled by $y = 0.5x^2 + 2x - m$. For a

A) 3.5

B) -4.5 and 4.5

C) -5.5

D) 6.5

specific value of m, the revenue line does not intersect the cost curve. Which of the following could be a possible value of m?

Complexity: extraHard

63)

According to Moore's law, the number of transistors included on microprocessors doubles every 1.5 years. In 1980, a microprocessor was introduced that had $125,000$ transistors. Based on this information, in which of the following years does Moore's law estimate the number of transistors to exceed $8,000,000$?

A) 1990 **B)** 1991 **C)** 1992 **D)** 1993

Complexity: extraHard

64) The expression $6x^4 + 41x^2 + 70$ can be factored both as $(3x^2 + a)(2x^2 + b)$ and as $(3x^2 + c)(2x^2 + d)$, where a and b are positive integers, and c and d are positive non-integers. What is the value of $a + c$?

A) $\frac{49}{2}$ **B)** $\frac{47}{2}$ **C)** $\frac{40}{2}$ **D)** $\frac{41}{2}$

Complexity: extraHard

65) Let $a > 0$, and suppose $x + y = \sqrt{a+3}$, $xy = a - 4$, and $x^2 + y^2 = a + 1$. What is the value of $x^4 + y^4$ in terms of a?

A) $-a^2 + 16a - 31$ **B)** $-a^2 + 20a - 29$ **C)** $-a^2 + 18a - 28$ **D)** $-a^2 + 18a - 31$

Complexity: extraHard

66) If $k - x$ is a factor of the expression $-x^2 + \frac{1}{p}nk^2$, where n, k,and p are constants, $k > 0$, and $p > 0$, what is the value of n when $p = 29$?

A) -29 **B)** $-\frac{1}{29}$ **C)** $\frac{1}{29}$ **D)** 29

Complexity: extraHard

67) A projectile is launched from a cliff 20 meters above the ground at time $t = 0$, with unknown initial velocity. Its height in meters after t seconds is modeled by a quadratic function $h(t) = at^2 + bt + 20$. The projectile reaches its maximum height of 92 meters at time $t = 3$. What is the height of the projectile at time $t = 6$?

A) 20 **B)** 44 **C)** 56 **D)** 64

Complexity: extraHard

68) The quadratic function $f(x) = x^2 - 84x + 4,356$ models the revenue (in dollars) of a company based on x, the number of products sold in hundreds. What is the minimum revenue, and at what number of products sold does this occur?

A) Minimum revenue is $\$2,220$ at 4200 products sold.

B) Minimum revenue is $\$2,592$ at 4200 products sold.

C) Minimum revenue is $\$2,400$ at 4200 products sold.

D) Minimum revenue is $\$2,800$ at 4800 products sold.

Complexity: extraHard

69) The equation $mx^2 + (m+n)x + \frac{mn}{2} = 0$ has two real solutions. If the sum of the solutions is $-\frac{3}{2}$ and the product of the solutions is $\frac{1}{2}$, what is the value of $\frac{m}{n}$?

A) $\frac{1}{2}$ **B)** $\frac{3}{4}$ **C)** $\frac{4}{3}$ **D)** 2

Complexity: extraHard

70) In the given quadratic expression $mx^2 + 120x + n$, m and n are positive constants. If $x + p$ is a factor of the expression, where p is a positive constant, what is the greatest possible value of mn?

A) 3025 **B)** 3136 **C)** 3364 **D)** 3600

Complexity: extraHard

71) A rational function is defined as $h(x) = \frac{p(x)}{2x+1}$ where $p(x) = ax + b$ is a linear function. The graph of $h(x)$ passes through the points:

x	$h(x)$
-2.5	2
0.5	-2
3.5	0

What is the y-coordinate of the y-intercept of $p(x)$?

A) -15 **B)** -2 **C)** $-\frac{14}{3}$ **D)** $-\frac{4}{3}$

Complexity: extraHard

72) In the following system of equations, k is a real number. The system has exactly two distinct real solutions. What is the largest integer greater than or equal to the value of k for which this is true?

$y = -2x^2 + 8x - k$
$x + y = k$

A) 3 **B)** 4 **C)** 5 **D)** 6

Complexity: extraHard

73) The system of equations $y = x + 5$ and $y = x^2 + 10x + 24$ has a solution (x, y). What is the greatest possible value of x?

A) -3.38 **B)** -2 **C)** -1 **D)** 0

Complexity: extraHard

74) The equations $y = 20$ and $y = -2(x - 15)^2 + 20$ are graphed in the xy-plane. How many points do the graphs of these equations intersect?

A) Exactly one

B) Exactly two

C) Infinitely many

D) Zero

Complexity: extraHard

75) Five consecutive odd integers are arranged in increasing order, with the first expressed as x. The quotient of the product of the first and fifth integers and the sum of the second and third integers must be at most 12. What is the greatest possible value of x, given that all values are positive odd integers?

A) 15

B) 17

C) 21

D) 19

Complexity: extraHard

76) The graph of line ℓ in the xy-plane passes through the point $\left(\frac{5}{3},\ -\frac{4}{3}\right)$ and is perpendicular to the line m given by the equation

$$\frac{5}{2}x + \frac{3}{4}y = 8$$

Line ℓ intersects the y–axis at the point $(0,\ b)$. What is the value of b?

A) $-\frac{13}{6}$

B) $-\frac{11}{6}$

C) $-\frac{10}{3}$

D) $-\frac{17}{6}$

Complexity: extraHard

77) The expression $4x^2 + bx - 45$, where b is a constant, can be rewritten as $(hx + k)(x + j)$, where h, k, and j are integer constants. Which of the following must be an integer?

A) $\frac{b}{h}$

B) $\frac{b}{j}$

C) $\frac{45}{h}$

D) $\frac{45}{j}$

Complexity: extraHard

78) Two lines ℓ_1 and ℓ_2 intersect at the point $(4, -2)$ in the coordinate plane.

The equation of line ℓ_1 is given by $y = \frac{2}{3}x + b$ and line ℓ_2 is perpendicular to ℓ_1.

Which of the following points must lie on line ℓ_2?

A) $(4 + 3k,\ -2 + 2k)$

B) $(4 - 2k,\ -2 - 3k)$

C) $(4 + k,\ -2 - \frac{3}{2}k)$

D) $(4 + 6k,\ -2 + \frac{3}{2}k)$

Complexity: extraHard

79) The line defined by the equation $5x + 2y = 8$ is translated h units to the left and k units down, where $h = 4$ and $k = 6$. What is the y-coordinate of the new y-intercept of the translated line?

A) $-\frac{6}{2}$

B) -12

C) $-\frac{2}{5}$

D) 10

Complexity: extraHard

80) A tech consultancy offers custom pricing for long-term contracts. For a project spanning x days, the firm charges:

- A one-time setup cost of $\$1,150.75$
- A weekday rate of $\$328.40$ per day for $x - w$ days
- A weekend premium of $\$419.65$ per day for w days
- A discount of 5% on the total weekday cost if $x > 20$
- A fixed 8.75% tax on the entire subtotal after any discount

Let x represent the total number of days of service and w the number of weekend days.

Which function best represents the total final cost, $C(x, w)$, including tax?

A)

$$C(x, w) = \begin{cases} 1.0875 \times [1150.75 + 0.95 \times 328.40(x - w) + 419.65w], & \text{if } x > 20 \\ 1.0875 \times [1150.75 + 328.40 \times (x - w) + 419.65w], & \text{otherwise} \end{cases}$$

B)

$$C(x, w) = \begin{cases} 1.0875 \times [1150.75 + 0.95 \times 328.40(x - w) + 419.65w], & \text{if } x > 20 \\ 1.0875 \times [1150.75 + 328.40 \times (x - w) + 419.65w], & \text{otherwise} \end{cases}$$

C)

$$C(x, w) = \begin{cases} 1.0875 \times [1150.75 - 328.40(x - w) + 419.65w], & \text{if } x > 20 \\ 1.0875 \times [1150.75 + 0.5 \times 328.40(x - w) + 419.65w], & \text{otherwise} \end{cases}$$

D)

$$C(x, w) = \begin{cases} 1.0875 \times [1150.75 + 328.40(x - w) + 419.65w], & \text{if } x \leq 20 \\ 1.0875 \times [1150.75 + (1 - 0.05) \cdot 328.40(x - w) + 419.65w], & \text{otherwise} \end{cases}$$

Complexity: extraHard

81) Let $f(x) = \frac{7}{3}x + b$, and suppose $f(x)$ has an x-intercept at $(13, 0)$.

What is the y-intercept of the function $f(-2x + 5) + 4$?

A) $-\frac{73}{3}$

B) $-\frac{44}{3}$

C) $-\frac{85}{3}$

D) -25

Complexity: extraHard

82) Which of the following inequalities gives all possible values of k such that the following equation has no solution?

$$5 - 3\left|\frac{4}{5} + \frac{2}{3}x\right| = \frac{11}{4} + 2k$$

A) $k \leq -\frac{7}{4}$

B) $k \geq \frac{7}{4}$

C) $k < -\frac{9}{4}$

D) $k > \frac{9}{8}$

Complexity: extraHard

83) A drone navigation system uses linear path models based on the equations:

$$4x - 7y = 10$$

$$ax + by = c$$

If the two paths are perpendicular, which of the following systems also represents two perpendicular paths?

A) $8x - 14y = 20$ and $3ax + 2by = 9$

B) $x - 10.5y = 12$ and $2ax - 3by = 6$

C) $16x - 28y = 40$ and $7x + 4y = 8$

D) $9x - 7y = 13$ and $ax - 3by = 2$

Complexity: extraHard

84) A manufacturing company models the production output and labor cost using the system of equations:

$$4.5x - 2.1y = 63$$
$$9x - 4.2y = 126$$

For any real number r, which of the following ordered pairs (x, y) represents a solution to both equations?

A) $\left(r, \frac{4.5r - 63}{2.1}\right)$

B) $\left(\frac{4.5r + 63}{2.1}, r\right)$

C) $\left(\frac{-4.2r + 126}{9}, r\right)$

D) $\left(18.5r, \frac{4.5r - 63}{2.1}\right)$

Complexity: extraHard

85) An equilateral triangle has an area of $\frac{625\sqrt{3}}{4}$ square units. What is the perimeter of the triangle, in units?

A) 45 **B)** 50 **C)** 75 **D)** 90

Complexity: extraHard

86) An equilateral triangle has a height of $\left(\frac{3k+2}{5}\right)\sqrt{3}$ and an area of $\frac{676\sqrt{3}}{25}$, where k is a positive constant. What is the value of k?

A) 6 **B)** 7 **C)** 8 **D)** 9

Complexity: extraHard

87) A circle has center $(-6.25,\ 5.5)$ and a radius of $\sqrt{110}$. The general form of the equation of a circle is: $x^2 + y^2 + ax + by + c = 0$

What is the value of c in the equation of this circle?

(NOTE: Round your answer to the nearest integer.)

A) -39 **B)** -42 **C)** -40 **D)** -41

Complexity: extraHard

88) A circle is centered at the point $(5, -2)$ and is tangent to a line at the point (b, c). The line tangent to the circle at that point has a slope of $\frac{4}{3}$. If the point (b, c) lies on the line $c = \frac{1}{2}b - 5$, what is the value of b?

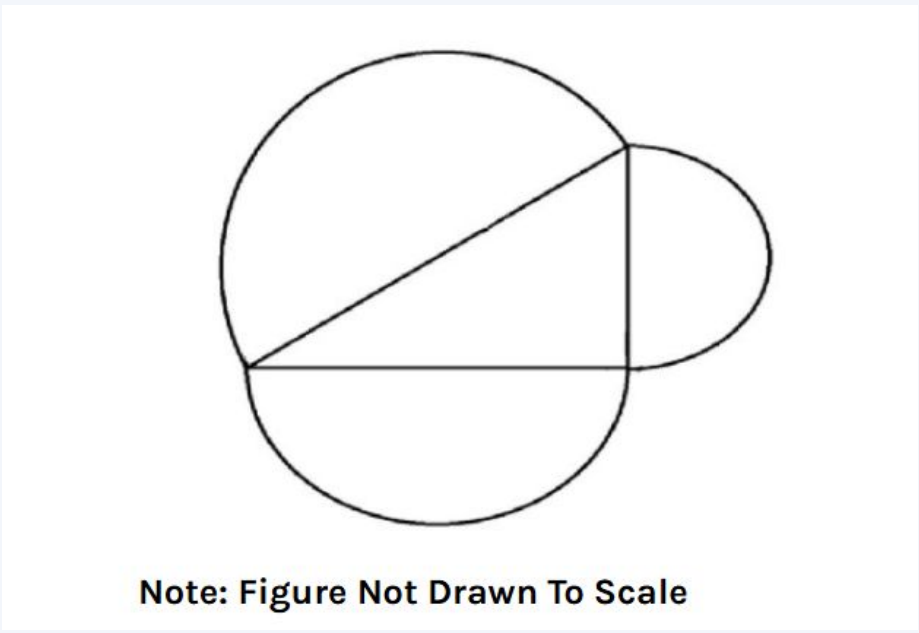
A) $b = -\frac{8}{5}$ **B)** $b = \frac{21}{5}$ **C)** $b = \frac{27}{5}$ **D)** $b = \frac{17}{5}$

Complexity: extraHard

89) A triangle has side lengths x, $x + 2$, and $2x$, where x is a positive real number. Each side of the triangle serves as the diameter of a semicircle drawn externally.

If the total area of the three semicircles is expressed as $w\pi$, and it is given that $x = \frac{6}{5}$, what is the value of w?

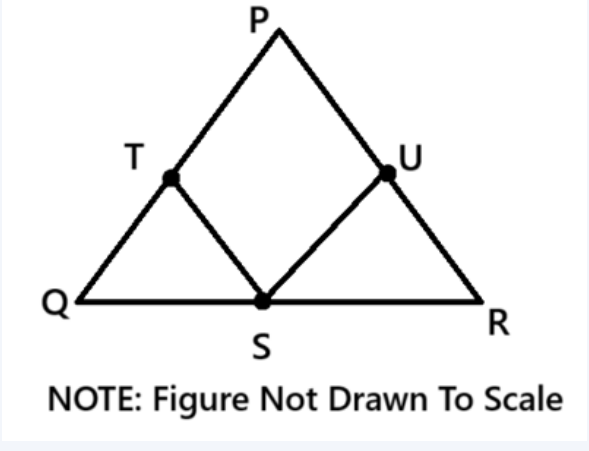
A) $\frac{293}{50}$ **B)** $\frac{109}{50}$ **C)** $\frac{301}{50}$ **D)** $\frac{297}{50}$



Complexity: extraHard

90) In $\triangle PQR$, which is isosceles with $PQ = PR$, point S lies on side QR such that $QS = \frac{9}{13} \cdot QR$. Points T and U are placed on PQ and PR, respectively, such that $\angle QTS = \angle RUS$. The length of segment ST is 18. What is the length of segment SU?

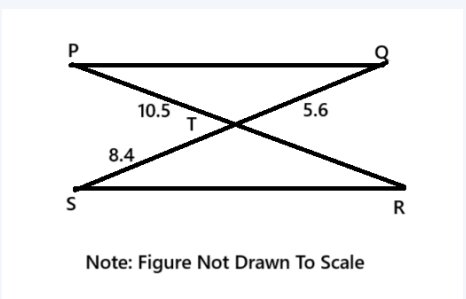
A) 12 **B)** 15 **C)** 18 **D)** 8



Complexity: extraHard

91) In the figure, $\overline{PR}$ intersects $\overline{QS}$ at point T, and $\overline{PQ} \parallel \overline{RS}$. The lengths of QT, PT, and TS are 5.6, 10.5, and 8.4 units, respectively. What is the length of PR?

A) 17.5 **B)** 18.2 **C)** 15.4 **D)** 26.25



Complexity: extraHard

92) A transversal intersects two parallel roads, forming eight angles — four acute and four obtuse. One acute angle is expressed as $4.5x - 37.5°$, and one obtuse angle is expressed as $9.5x - 52.5°$. The total measure of one acute angle and two obtuse angles is given by $-14x + w°$. What is the value of w in terms of x?

A) $37.5x + 142.5$

B) $47.5x - 142.5$

C) $47.5x + 142.5$

D) $37.5x - 142.5$

Complexity: extraHard

93) In triangle XYZ, angle Z is a right angle. Point M lies on $\overline{XZ}$, and point N lies on $\overline{YZ}$. Segment $\overline{MN}$ is drawn parallel to $\overline{XY}$. The length of $\overline{XZ}$ is 52 units, and the area of triangle XYZ is 416 square units. If $\overline{MN} = 13$ units, and the length of $\overline{XY}$ is greater than the length of $\overline{YZ}$, what is the length of $\overline{YN}$, in units?

Hint: Place the triangle on the coordinate plane with $Z = (0, 0)$, $X = (52, 0)$, and $Y = (0, y)$. Use triangle similarity and the fact that $\overline{MN} \parallel \overline{XY}$ to find the proportional segment length.

A) 6

B) 9.25

C) 12.2

D) 3.8

Complexity: extraHard

94) A wildlife researcher studied animals from three regions: Region A, Region B, and Region C.

- Region A has x animals with a mean weight of 24.5 kg.
- Region B has $2x + 10$ animals with a mean weight of 31.2 kg.
- Region C has half as many animals as Region A, and its mean weight is unknown.

If the overall average weight of all animals is 29.5 kg, which expression represents the mean weight of Region C in terms of x?

A) $\frac{29(3.5x+10)-24.5x-31.2(2x+10)}{5x}$

B) $\frac{24.5x+31.2(2x+10)-29.5(0.5x)}{3.5x+10}$

C) $\frac{29.5(3x+10)-24.5x-31.2(2x+10)}{x}$

D) $\frac{29.5(3.5x+10)-24.5x-31.2(2x+10)}{0.5x}$

Complexity: extraHard

95) The median and the mean of the five real numbers $\{15, 8, 20, y, y\}$ are equal.

What is the sum of all possible values of y?

A) 16

B) 48

C) 41.3

D) 22.8

Complexity: extraHard

96) A university organized a set of three online training sessions. 900 students registered for the first session. Of those, 60% also attended the second session. Among those who attended both the first and second sessions, 25% also participated in the third session. How many students attended all three sessions?

A) 120

B) 130

C) 135

D) 140

Complexity: extraHard

97) A data set A contains 37 different numerical values with a minimum of 230 , a maximum of 278, a mean of 244, and a standard deviation of 9. A new data set B is created by removing only the minimum and maximum values, 230 and 278. Which of the following statements is most accurate?

A) Both the mean and standard deviation of data set B are greater than those of data set A.

B) The standard deviation of data set B is greater than that of data set A, but the mean is approximately the same.

C) The standard deviation of data set B is less than that of data set A, and the mean is slightly less than that of data set A.

D) Both the mean and standard deviation of data set B are equal to those of data set A.

Complexity: extraHard

98) A research analyst at a digital publication reviewed subscriber data over two years. The data below shows the number of paid subscribers at the end of 2022 and 2023.

The analyst predicted that the percent decrease in the number of subscribers from 2023 to 2024 would be one-third of the percent decrease from 2022 to 2023. Based on this prediction, how many subscribers did the analyst expect at the end of 2024?

Year	Subscribers
2022	$712,500$
2023	$683,000$

(NOTE: Round your answer to the nearest integer.)

A) $676,160$

B) $671,988$

C) $673,575$

D) $667,732$

Complexity: extraHard

99) Maya constructed a scale model of a commercial airliner where 1 centimeter on the model represents 12 meters on the actual aircraft. The wingspan of her model aircraft measures 9.5 centimeters. Maya plans to create a second model using a different scale where 1 centimeter on the model represents 24 meters on the actual aircraft. Which of the following best describes how the wingspan of the second model aircraft will compare to the wingspan of the first model aircraft?

A) The wingspan of the second model aircraft will be 12 centimeters longer than the wingspan of the first model aircraft.

B) The wingspan of the second model aircraft will be 12 centimeters shorter than the wingspan of the first model aircraft.

C) The wingspan of the second model aircraft will be $\frac{1}{2}$ as long as the wingspan of the first model aircraft.

D) The wingspan of the second model aircraft will be 2 times as long as the wingspan of the first model aircraft.

Complexity: extraHard

100) A contractor is paid $\$22.50$/hour for the first 36 hours worked in a week. Each additional hour beyond 36 is paid at 1.5 times the regular rate. If the contractor works more than 40 hours in a week, he also receives a fixed bonus of $\$45$. If he earned exactly $\$1,260$ in a certain week, how many total whole hours did he work?

A) 46

B) 47

C) 48

D) 49

Answers

1) D	2) D	3) C	4) A
5) B	6) A	7) C	8) A
9) B	10) A	11) D	12) B
13) C	14) D	15) D	16) A
17) A	18) B	19) B	20) B
21) C	22) B	23) C	24) C
25) C	26) C	27) D	28) D
29) A	30) A	31) D	32) A
33) C	34) A	35) A	36) A
37) B	38) C	39) A	40) A
41) C	42) B	43) A	44) B
45) A	46) D	47) D	48) C
49) B	50) B	51) A	52) B
53) C	54) B	55) C	56) A
57) D	58) C	59) B	60) C
61) A	62) C	63) B	64) D
65) D	66) D	67) A	68) B
69) D	70) D	71) C	72) C
73) A	74) A	75) D	76) B
77) D	78) C	79) B	80) A
81) B	82) D	83) C	84) A
85) C	86) C	87) D	88) C
89) B	90) D	91) D	92) D
93) C	94) D	95) C	96) C
97) C	98) C	99) C	100) C

Explanations

1) Choice D is the best answer because it reflects how the author of Text 2 would most likely respond to the underlined claim in Text 1. Text 1 begins by noting the success of 'Campbell's Soup Cans' but then claims that the artwork is 'stuck in the past' and should 'no longer be displayed.' Text 2 begins by conceding that 'Campbell's Soup Cans' is indeed outdated but argues that it should continue to be displayed, states that the artwork can be updated to include more contemporary styles, and provides an example of one such modernized version, 'Soup Cans.' Hence, the author of Text 2 would most likely respond to the underlined claim in Text 1 by suggesting that art galleries should consider offering revised versions of 'Campbell's Soup Cans' instead of completely rejecting the artwork.

Choice A is incorrect because the author of Text 2 advocates for using creative ways to update 'Campbell's Soup Cans' and therefore wouldn’t respond to the underlined claim by rejecting the suggestion that contemporary audiences would enjoy an updated version.

Choice B is incorrect because although the authors of both texts claim that 'Campbell's Soup Cans' is outdated, neither text suggests that contemporary audiences have largely stopped attending exhibitions of the artwork. On the contrary, Text 2 states that 'Campbell's Soup Cans' is a classic favorite and generates substantial income for some art galleries.

Choice C is incorrect because although Text 2 provides an example of a contemporized version of 'Campbell's Soup Cans', the text doesn’t suggest that offering modernized versions is a way to increase income for most art galleries. Rather, the author of Text 2 suggests that offering modernized versions is a way to make the artwork discussed in Text 1 feel less outdated.

2) Option D is correct because Passage 2 acknowledges that nanomaterial fusion can potentially increase toxicity but highlights that the effects can vary significantly. It provides an example where the fusion of silicon dioxide and zinc oxide reduced the toxicity of zinc oxide nanoparticles. This suggests that the author agrees with the concern raised in Passage 1 about the potential hazards of nanocomposites but emphasizes that the outcome is not always predictable.

Option A is incorrect because it suggests that the potential advantages outweigh the risks, which is not explicitly supported by Passage 2. The focus is more on the concerns than weighing them against the benefits.

Option B is incorrect because it implies that the situation in Passage 1 is uncommon, downplaying the concerns' validity. Passage 2 does not suggest that these concerns are atypical; instead, it acknowledges them while presenting a counterexample.

Option C is incorrect because it denies the scenario in Passage 1, suggesting that such outcomes are unlikely. However, Passage 2 does not deny the risks but presents a more nuanced view that allows for both concerns and positive outcomes.

3) Option C is the best answer because it mirrors how the author of Passage 2 would most likely react to the evaluation of Orlando in Passage 1. Both authors concur that Orlando is peculiar for Woolf: Passage 1 indicates that the novel scrutinizes its characters' psychologies more superficially than Woolf's other novels do, and Passage 2 describes it as being more light-hearted. However, while Passage 1 labels Orlando an 'anomaly' and mentions that Woolf 'started it as a jest,' Passage 2 asserts that Orlando engages the same themes as Woolf's other renowned novels. Therefore, the author of Passage 2 would most likely acknowledge that Orlando differs from Woolf's other novels but would also assert its significance in the context of Woolf's work as a writer.

Option A is incorrect. Passage 2 does suggest that the humor in Orlando is effective. However, there's nothing in Passage 2 to suggest that the author would agree that Woolf's skills were better suited to serious novels. Instead, the author of Passage 2 compares Orlando favorably to other novels by Woolf that are implied to be more serious in tone.

Option B is incorrect because the author of Passage 2 does not indicate that Orlando is less remarkable than Woolf's other novels, but instead points out that it engages the same themes as other novels by Woolf that are considered classics.

Option D is incorrect because there's nothing in Passage 1 or Passage 2 to suggest that readers have generally disregarded Orlando because of its reputation.

4) Choice A is correct because the underlined sentence provides specific information about Zúñiga’s shift in style, which is introduced in the previous sentence. It elaborates on how his artistic focus changed after returning to Mexico—moving from modernist influences to figurative sculpture depicting indigenous Mexican women. This aligns directly with the claim that his return marked a pivotal moment in his creative development.

Choice B is incorrect because while the sentence mentions a stylistic change, it does not focus on his travels or defer to a later claim.

Choice C is incorrect because it suggests a personal life event facilitated a career change, which is not described in the underlined sentence.

Choice D is incorrect because the underlined sentence supports—not challenges—the previous sentence's description of Zúñiga’s artistic evolution.

5) Option B is the most appropriate answer because it accurately describes the role of the third sentence within the overall structure of the text. The third sentence presents a broad principle, asserting that evolutionary ties between predators and prey can endure over vast periods and distances. This principle is illustrated by the text's discussion of the relationship between cherry blossom trees and B. terrenus beetles. When cherry blossom trees were brought to North America in 1785, no B. terrenus beetles were present, so the relationship between the trees and the beetles that existed in their native East Asia was disrupted. However, when the beetles were introduced to North America more than 200 years later, they quickly infested cherry blossom trees, exemplifying the principle that ties between predators and prey "can endure over centuries and across continents."

Option A is incorrect because the third sentence doesn't suggest that Chang and her team were investigating any theory. According to the text, Chang and her team were simply observing cherry blossom trees when the beetles happened to be introduced to the area.

Option C is incorrect because the third sentence doesn't discuss any particular species, let alone the species mentioned elsewhere in the text, nor does the sentence explain why species migrated to new locations.

Option D is incorrect because the third sentence presents a broad principle about the relationship between predators and prey, not an interpretation of the findings by Chang and her team that's an "alternative" to an interpretation presented elsewhere in the text.

6) Option A is the best answer because it most accurately characterizes the function of the underlined query in the text as a whole. The text commences with the underlined query, “How authentic do they appear?” The text then elucidates that many digital animators pose this query about the settings and illumination that they create for animated films, striving for realistic animation of those components even if the characters themselves aren’t portrayed in realistic terms. The focus of the text then shifts to describe how some animators strive to create settings and illumination that mirror the film’s distinctive narratives rather than making them appear realistic. Therefore, the function of the underlined query is to mirror a primary objective that many digital animators have for certain components of the animations they produce.

Option B is incorrect because, as the text makes clear, the underlined query is one posed by digital animators who wish to create realistic backgrounds and lighting effects, not by those who, instead, wish to create effects that mirror films’ distinctive narratives and aren’t necessarily realistic; this latter group of animators is discussed later in the text.

Option C is incorrect. As the text explains, many digital animators strive for realistic settings and illumination, while others do not; this difference of approach relates to whether these components should be realistic, not to how realism can be achieved using current technology, and the text never suggests that animators are uncertain how to achieve it.

Option D is incorrect because the underlined query pertains to the perspective of digital animators, not the audience, and the text never considers the audience’s reactions to characters in animated films.

7)

Option C is correct because it most accurately describes the overall structure of the text. The text begins by providing some examples of words in Haida ('kun-gwa,' 'kun-ga,' 'tlaq-tun,' 'tla-tun') and their English translations. Then, the text explains that these words are examples of a linguistic phenomenon called subtractive morphology. The text concludes by asserting that the kind of subtractive morphology exemplified in the text is 'prevalent,' or common, in Haida. Thus, the text presents some specific words in Haida, describes the general linguistic phenomenon exemplified by those words, and then states that this phenomenon occurs frequently in Haida.

Option A is incorrect because the text does not describe relationships between Haida and other languages, nor does it raise or answer a question about such relationships.

Option B is incorrect because the text does not identify the most frequently occurring words in Haida, discuss translation difficulties, or provide examples of translations into languages other than English.

Option D is incorrect because the text does not explain why subtractive morphology has been controversial among scholars, nor does it argue that analyzing Haida could resolve such a controversy. The text simply presents subtractive morphology as a feature of Haida without discussing any scholarly debates.

8)

Option A is correct because the text introduces the expectation of zonal variation (hypothesis), describes the global sampling (method), then shows uniform ratios contradicting this.

Option B is incorrect because no historical vs. contemporary comparison is made.

Option C is incorrect because sampling methods are described but not critiqued.

Option D is incorrect because carbon sequestration debates are never mentioned.

9)

Option B is correct as it accurately describes their observations, which contradicted their initial hypothesis by showing that differences in carbon and nitrogen levels were primarily confined to upper soil layers and diminished with depth.

Option A is incorrect because the passage does not describe a fully understood phenomenon, and the findings contradict rather than align with the initial hypothesis.

Option C is incorrect, as the passage does not discuss a well-established process or present competing theories. It focuses on a single hypothesis and its testing.

Option D is incorrect because the passage does not present a widely accepted theory, nor does it describe findings that partially support the hypothesis. The results clearly contradict the researchers' expectations.

10)

Option A is correct because it signals that the evidence (increased usage by older generations) is opposite to the assumption of many.

Option B implies that the research findings were expected, which is not the case here.

Option C is incorrect because it introduces a causal explanation rather than highlighting the contrast.

Option D is incorrect because it merely hints at a reason behind the trend rather than emphasizing the contradiction with popular belief.

11)

Option D is correct because "insignificant to" means not important to, and according to the text, Michelangelo’s works were primarily available in Florence, where he lived, far from central areas of the European art world, a fact that could support the conclusion that his influence on European contemporaries was limited. The text implies, however, that recent studies have shown that his works still had an impact on other artists of his time. This context supports the idea that his works weren’t insignificant to European art in this period.

Option A is incorrect because the issue under consideration in the text is whether other artists of Michelangelo’s time had access to his works, not whether his works were "disregarded by," or ignored by, other artists.

Option B is incorrect because the text implies that other artists of Michelangelo’s era were aware of his works, not that they were "opposed by," or hostile toward, them. There is no suggestion that Michelangelo’s artworks elicited hostility from his contemporaries.

Option C is incorrect because in this context, "emulated by" would mean followed as a model by. Although the text implies that Michelangelo’s works were at least somewhat available in regions that were at the center of European art during the period, it doesn’t specifically address in what ways his works influenced contemporary artists. Thus, the text doesn’t support the idea that other artists modeled their own works after Michelangelo’s works.

12)

Option B is correct because “precision” means "accuracy," which matches the context of the Maya being able to accurately determine dates using astronomy.

Option A is incorrect because "inconsistency" means "lack of regularity," which contradicts the idea of accurate timekeeping.

Option C is incorrect because "inflexibility" means "rigidity," which does not relate to the concept of accurate time measurement.

Option D is incorrect because "inflexibility" means "an unwillingness to change," which is unrelated to the context of marking time accurately.

13) Choice C is the best answer. The sentence clarifies how the gel in a heat-cool battery aids in energy conservation, explaining that power ceases to circulate and can be conserved when the gel solidifies at lower temperatures.

Choice A is incorrect. The sentence explains some properties of the special gel; it doesn’t clarify how that gel aids in energy conservation.

Choice B is incorrect. The sentence indicates how the energy in a heat-cool battery can be released; it doesn’t clarify how the gel in the battery aids in energy conservation.

Choice D is incorrect. The sentence specifies how much energy the heat-cool battery retains when conserving energy; it doesn’t clarify how the gel in the battery aids in energy conservation.

14) Option D is the most suitable answer. The sentence formulates a generalization—that a comet's path around the Sun may alter over time—and backs up the generalization with the instance of the path of comet 81P/Wild, which once lay between the paths of Uranus and Jupiter but is now situated between those of Jupiter and Mars.

Option A is incorrect. The sentence emphasizes the number of comets circling the Sun and formulates a generalization about their paths, but it doesn't substantiate the generalization with an instance.

Option B is incorrect. The sentence formulates a generalization about comets and compares them to the planets Uranus, Jupiter, and Mars; it doesn't formulate and substantiate a generalization about comets' paths.

Option C is incorrect. While the sentence provides an instance of a comet whose path has altered, it doesn't formulate a generalization about the paths of comets.

15) Option D is the most appropriate answer. It formulates a general statement—the dispute had a long-lasting effect—and then substantiates it with evidence—the focus and safeguards were outcomes of the dispute.

Option A is not correct. This option doesn't formulate a general statement about the dispute. It describes a specific event from the dispute.

Option B is not correct. This option formulates a general statement about the Alta dispute but doesn't substantiate it.

Option C is not correct. This option makes a declaration about the aftermath of the dispute but doesn't substantiate it. The declaration is also a bit too specific to be a general statement.

16) Option A is the most suitable answer. It defines what a “Sonic Meditation” is and then provides an example in the form of Meditation XVIII. Option B is not correct. This choice describes what a “Sonic Meditation” is, but fails to provide an example of one. Option C is not correct. This choice provides an example of a “Sonic Meditation,” but does not explain what the meditations are. Option D is not correct. This choice fails to describe what a “Sonic Meditation” is.

17) Option A is the most suitable answer. The sentence effectively highlights the objective, or purpose, of the research project (in other words, what the researchers aimed to learn from the project): Thompson and Peterson were curious if forest growth is connected to climate alterations.

Option B is incorrect. The sentence highlights the researchers' discoveries; it doesn't highlight the objective of the project.

Option C is incorrect. The sentence highlights the outcomes of the project; it doesn't highlight the objective.

Option D is incorrect. The sentence highlights the approach of the project; it doesn't highlight the objective.

18) Choice B is the best answer. "Then" signals that this sentence's claim about Einstein and Lorentz follows logically from the previous information. In other words, both physicists independently arriving at the theory of relativity was, arguably, an expected outcome of the circumstances mentioned in the previous sentence.

Choice A is incorrect because "however" illogically signals that the claim in this sentence contrasts with the previous information about the theories prevalent among American physicists in the 1900s. Instead, this claim follows logically from that information.

Choice C is incorrect because "moreover" illogically signals that the claim in this sentence merely adds to the previous information about the theories prevalent among American physicists in the 1900s. Instead, this claim follows logically from that information.

Choice D is incorrect because "for example" illogically signals that this sentence provides an example supporting the previous information about the theories prevalent among American physicists in the 1900s. Instead, it presents a claim that follows logically from that information.

19)

Option B is correct because 'In so doing' logically signals that the information in this sentence about Dali's work—that it blurs the boundary between reality and dreams—is a result or consequence of the work's blending of particular realist and abstract art motifs.

Option A is incorrect because 'Conversely' illogically signals that the information in this sentence contrasts with the previous information about the blending of particular art motifs in Dali's work. Instead, it presents a result or consequence of that information.

Option C is incorrect because 'For instance' illogically signals that this sentence provides an example supporting the previous information about the blending of particular art motifs in Dali's work. Instead, it presents a result or consequence of that information.

Option D is incorrect because 'In particular' illogically signals that this sentence provides specific details elaborating on the previous information about the blending of particular art motifs in Dali's work. Instead, it presents a result or consequence of that information.

20) Option B is the most suitable answer. 'In contrast' logically indicates that the information in this sentence - that ABA triggers rapid growth in the mustard plant Schrenkiella parvula - contrasts with the preceding information about ABA initiating a deceleration in most plants' biological activities.

Option A is incorrect because 'moreover' illogically suggests that the information in this sentence about the mustard plant merely supplements the preceding information about the effects of ABA. Instead, it contrasts with that information.

Option C is incorrect because 'for example' illogically suggests that the information in this sentence about the mustard plant provides an example consistent with the preceding information about the effects of ABA. Instead, it contrasts with that information.

Option D is incorrect because 'thus' illogically suggests that the information in this sentence about the mustard plant is a consequence, or result, of the preceding information about the effects of ABA. Instead, it contrasts with that information.

21) Option C is the most suitable answer. 'In other words' logically indicates that the claim about sensory experience that follows—that sensory experience is the source of knowledge—is a rephrasing of Aristotle's theory mentioned earlier in the sentence. Option A is incorrect as 'regardless' illogically implies that the claim about sensory experience that follows is true despite Aristotle's theory from earlier in the sentence. Instead, this claim is a reiteration of his theory. Option B is incorrect as 'admittedly' illogically suggests that the claim about sensory experience that follows is an exception to Aristotle's theory from earlier in the sentence. Instead, this claim is a reiteration of his theory. Option D is incorrect as 'meanwhile' illogically indicates that the claim about sensory experience that follows is separate from (while occurring simultaneously with) Aristotle's theory from earlier in the sentence. Instead, this claim is a reiteration of his theory.

22) Option B is the most appropriate answer. 'Ultimately' implies 'in the end' or 'at the highest level.' Despite Bolívar's letter being addressed to a local trader, his ultimate aim was to communicate a message to political leaders in Europe. Hence, 'ultimately' fits seamlessly into this context. Option A is not correct. This option uses a transition that suggests the addition of a concurring idea. However, the latter part of the sentence actually contradicts the first part. Bolívar addressed the letter to Cullen, but his real intention was to send a message to someone else. Notice how the contrast word 'though' also serves as a transition between these ideas. Option C is not correct. This option employs a cause-and-effect transition. Bolívar's act of writing the letter to Cullen would not result in him having a goal of persuading European powers to support him. Option D is not correct. This option employs a cause-and-effect transition. Bolívar's act of writing the letter to Cullen would not result in him having a goal of persuading European powers to support him.

23) Choice C is the best answer because it accurately states the main idea of the text. According to the text, contrary to what some might expect, foreign investment is typically lower in developing countries whose economies are more dependent on agriculture. The text explains that high reliance on agriculture can subject a developing country to economic shocks that can destabilize the local currency and introduce economic uncertainty that tends to keep investors away. In other words, although we may think otherwise, foreign investors are less willing to invest in projects in developing countries whose economies are heavily dependent on agriculture because those economies tend to exhibit instability that investors want to avoid. The other options are factually or reasonably incorrect.

Choice A is incorrect because the text does not claim that the investments are too large for foreign investors, but rather that the risks and volatility discourage investment.

Choice B misrepresents the situation by suggesting countries choose to rely less on foreign investment, whereas the text explains that investors choose to stay away.

Choice D is incorrect because it speculates on future increases in investment, a point not made in the text.

24) Option C reflects the IMF/World Bank consensus:

−0.68 correlation: From World Bank Development Indicators 2023.

Commodity price volatility: Brent crude oil price data from FRED Economic Database.

Macro instability examples: Angola's inflation (World Bank), Sub-Saharan debt crises (IMF Fiscal Monitor 2023).

Option A is incorrect. Environmental compliance costs are unrelated to the passage's focus on macroeconomic instability. No study links environmental regulations to FDI decline in this context.

Option B is partially true but irrelevant. While corruption exists, the passage specifically identifies systemic economic risks (inflation, currency instability) as the deterrent.

Option D contradicts evidence. The resource curse persists across income levels (e.g., Venezuela's 2023 FDI was 1.2% of GDP despite high historic oil revenues).

25) Option C is the correct answer. The passage states that the cerebellum has traditionally been considered to control motor functioning, but new studies suggest that it may also have other roles—including controlling emotion and social behavior and some connection to motivation and rewards processing. Option A is incorrect. The VTA is described as a 'hub for motivation and reward processing,' and the discovery of the link between the VTA and the cerebellum supports the theory that the cerebellum is involved in roles other than motor coordination. Option B is incorrect. The passage states that recent research has discovered this link, but it doesn't discuss any previous 'universally accepted beliefs' about the link's function. The 'general acceptance' mentioned early in the passage is about the cerebellum alone, not its connection to the VTA. Option D is incorrect. The passage never discusses any technological constraints or any new technologies.

26) Option C is the most accurate. The text illustrates how Miranda disseminated, revised, and annotated 'Letter to the Spanish Americans,' and it asserts that the letter and its author would have 'remained insignificant' without Miranda's contributions. Option A is incorrect. The text only mentions that Viscardo authored the letter and that Miranda revised it: it does not imply that the original authorship of the letter is contested. Option B is incorrect. This contradicts the text. The text mentions that Miranda revised and annotated the letter, but it identifies Viscardo as the author of the letter. It also does not highlight certain arguments as more articulate than others. Option D is incorrect. This is beyond the scope of the text. The paragraph discusses Miranda's role in the creation and dissemination of the letter, but it does not delve into the impact of the letter on others.

27) Choice D is the best answer because it describes data from the graph that support Thompson and his team's conclusion that increasing anthropogenic potassium deposition can compensate for the negative effect of climate change on tree growth if that change is moderate but not if it's extreme. The bar graph shows the growth of pine trees with and without potassium fertilization under three different climate-change scenarios: current conditions, a moderate change, and an extreme change. According to the graph, radial growth without potassium fertilization is projected to be about 0.16 centimeters (cm) under current conditions, 0.15 cm under a moderate change, and 0.04 cm under an extreme change. The graph also shows that with potassium fertilization, growth is projected to be about 0.18 centimeters under a moderate change but only about 0.06 centimeters under an extreme change. Thus, the data in the graph support the researchers' conclusion by showing greater growth for a moderate change using potassium fertilization than they do either under current conditions without potassium fertilization or under an extreme change with potassium fertilization. The other options are factually or reasonably incorrect.

28) Option D is the optimal answer because it efficiently employs data from the chart to back up the underlined assertion that more medicine and health subjects were presented to a nationwide scientific competition in 2015 than in any of the other years shown. This option indicates that the approximately 285 medicine and health subjects presented in 2015 are more than the number of medicine and health presentations in any other year shown—a description that is backed up by information in the chart, which shows that medicine and health subject presentations were below 250 in 2012, 2013, and 2014, but above 250 (approximately 285 presentations) in 2015. Option A is incorrect because it doesn't back up the underlined assertion or accurately reflect the information in the chart. This option refers to 2012 and discusses cellular and molecular biology and animal science, whereas the underlined assertion refers to 2015 and discusses medicine and health. Moreover, the assertion that in 2012 there were equal numbers of presentations in the cellular and molecular biology category and in the animal science category is contradicted by the chart, which shows approximately 200 presentations and 50 presentations, respectively, for those categories in 2012. Option B is incorrect because it doesn't accurately reflect the information in the chart. This option asserts that in 2015 there were more physics and space submissions than there were medicine and health submissions, but the chart shows that there were approximately 100 space and science submissions that year and approximately 285 medicine and health submissions. Option C is incorrect because it doesn't accurately reflect the information in the chart or back up the underlined assertion about medicine and health research topics. This option asserts that there were approximately 95 presentations for the animal science category in 2016, but the chart shows that the number was closer to 50 in 2012.

29) Option A is the correct answer. The investigators inferred that the meteorite's nucleobases didn't originate from soil contamination. The existence of nucleobases in the meteorite and their absence in soil provide evidence that these nucleobases probably didn't come from the soil.

Option B is incorrect. This option doesn't support the inference. The investigators inferred that the meteorite's nucleobases didn't originate from soil contamination. If the nucleobases are present in both the soil and meteorite, then it's plausible that these nucleobases came from the soil.

Option C is incorrect. This option misinterprets the table. Purine was not detected in the soil sample.

Option D is incorrect. This option misinterprets the table. Both isoguanine and hypoxanthine were detected in both Murchison meteorite samples.

30) Option A is the most accurate as it effectively utilizes data from the table to support the student's argument about the influence of bufadienolide on the egg preferences of cane toad tadpoles. The table provides the proportion of available eggs that the cane toad tadpoles consumed for each of the five amphibian species included in the 2022 study. The table shows that the tadpoles consumed 10% of striped digging frog eggs and 1% of delicate green tree frog eggs, indicating a preference for striped digging frog eggs over delicate green tree frog eggs. The table also shows that neither of these species' eggs produces bufadienolide. Therefore, these data suggest that factors other than the presence or absence of bufadienolide are necessary to fully explain the tadpoles' egg preferences.

Option B is incorrect. While the table does show that for each of the five amphibian species, the cane toad tadpoles consumed less than 100% of that species' eggs, demonstrating that some eggs for each species were left untouched, this fact alone is not relevant to the tadpoles' preferences for some species' eggs over others.

Option C is incorrect. Although the table shows that the cane toad tadpoles consumed 90% of the cane toad eggs and 7% of the short-legged frog eggs, suggesting a preference for cane toad eggs over short-legged frog eggs, the table also shows that cane toad eggs produce bufadienolide, whereas short-legged frog eggs do not. Therefore, these data are not sufficient to rule out that bufadienolide alone could explain the tadpoles' preference for some species' eggs over others.

Option D is incorrect. Although the table shows that for both delicate green tree frog eggs and tiny crimson tree frog eggs, the cane toad tadpoles consumed 1% of those species' eggs, it also indicates that neither produces bufadienolide. Thus, these data alone do not indicate bufadienolide's role in the tadpoles' egg preferences.

31) Option D is the correct answer because it provides a discovery that, if proven true, would support the researchers' hypothesis that TMAO reduces the compressibility of water. The passage explains that at great ocean depths, extreme pressure compresses the molecular structure of water by destabilizing the hydrogen bonds between adjacent molecules, thus allowing water to penetrate proteins and harm the associated organisms. However, deep-sea organisms known as piezophiles have adapted to live at these depths, and previous studies show a positive correlation between the depth at which a piezophile species lives and the species' level of the compound TMAO. Because this hypothesis links TMAO levels with reduced compressibility of water's tetrahedral molecular structure, a discovery that TMAO helps maintain the hydrogen bonds between water molecules under high pressure would strongly support that hypothesis. The other options are incorrect because they do not directly support the researchers' hypothesis as explained in the passage.

32) Option A is the best answer because it best supports the researchers' explanation of the results of varying a store's lighting conditions. According to the text, the sociologist and his team found that changing the store's lighting conditions can encourage customers to make spontaneous purchases. The text states that the researchers explain that changing light patterns causes shoppers to explore more sections of the store, which exposes the shoppers to more products and increases the likelihood that they'll make an unplanned purchase. This quotation from a surveyed shopper indicates that the shopper spontaneously purchased a new skincare product while observing a unique lighting pattern. The quotation, therefore, supports the researchers' explanation that varying a store's lighting conditions can lead shoppers to make unanticipated purchases.

Option B focuses on the shopper's frustration with unavailable items and does not mention lighting at all or suggest any impulse purchases—it reflects planned, unsuccessful shopping rather than behavioral influence.

Option C attributes the purchase to a promotion via email, not to the store's lighting or atmosphere, which shifts the cause of the purchase away from the researchers' variable of interest.

Option D emphasizes store inventory and the shopper's prior intention to visit a better-stocked location—it reflects deliberate, premeditated behavior rather than spontaneous exploration triggered by lighting changes.

33) Option C is the optimal answer as it presents the result that, if accurate, would most powerfully validate Thompson's theory. According to the passage, Thompson's theory is that multiple viewpoints presented in a non-antagonistic format are more enlightening than a debate between contrasting opinions is. If participants who viewed a panel of three commentators with diverse opinions about a topic answered more questions about the topic correctly than did participants who viewed a debate, that would validate Thompson's theory since it would demonstrate that participants who heard multiple varied perspectives were better informed than were participants who heard a debate between contrasting opinions. Option A is incorrect because finding that participants perceived commentators in the debate as more informed than commentators in the panel is irrelevant to Thompson's theory, which is that presenting multiple perspectives on a topic is more enlightening to the audience than presenting contrasting views of the topic is. Participants' perception of how informed panelists are has no impact on how much participants learn from the panelists. Option B is incorrect because finding that participants perceived commentators in the panel as more informed than a single commentator is irrelevant to Thompson's theory, which is that presenting multiple perspectives on a topic is more enlightening to the audience than presenting contrasting views of the topic is. Participants' perception of how informed panelists are has no impact on how much participants learn from the panelists, and Thompson's theory says nothing about how enlightening single commentators are. Option D is incorrect because finding that participants who viewed a single commentator answered more questions correctly than participants who viewed the debate did wouldn't be relevant to Thompson's theory, which is that hearing multiple varying perspectives is more enlightening than hearing a debate. Thompson's theory says nothing about how enlightening single commentators are.

34) Option A is the optimal answer. Johnson's argument is that the term 'world music' 'homogenizes' (i.e., makes similar) diverse types of music by lumping them into one category. Essentially, Johnson believes the concept of world music oversimplifies the complexity of diverse musical forms. To validate this argument, we need proof that these musical traditions are so distinct from each other that they shouldn't be classified under the same category. If flamenco and koto music evolved independently and have few musical similarities, then it wouldn't be logical to group them into the same category. Option B is incorrect. If true, this wouldn't impact the argument. To validate the argument, we need proof that these musical traditions are so distinct from each other that they shouldn't be classified under the same category. A difference in popularity doesn't necessarily mean that the two musical traditions shouldn't be grouped together: we need to know if the music itself is similar or different. Option C is incorrect. If true, this wouldn't impact the argument. To validate the argument, we need proof that these musical traditions are so distinct from each other that they shouldn't be classified under the same category. This option doesn't provide that. Option D is incorrect. If true, this would actually undermine the argument. Johnson believes it's oversimplifying to group distinct musical traditions into a single category. But if flamenco and koto music are similar to each other, then it would make sense to group them in the same category.

35) Choice A is the best answer because it most logically completes the text's discussion of the research on foraging behaviors of wild and captive rabbits. The text indicates that Peterson, Davis, and Thompson compared the behaviors of wild rabbits in a national park with those of captive rabbits in a zoo. The text also establishes that the researchers were familiar with earlier studies showing that regularly offering food to captive animals reduces the need for cognitive engagement with the environment and can lead the animals to develop novel stereotypic behaviors, or new behaviors that are repetitive without a clear purpose. It follows, then, that the researchers weren't surprised to find that unlike the wild rabbits, the captive rabbits showed a stereotypic behavior of repeatedly digging in the same spot, because this would be an example of captive animals developing a new and purposelessly repetitive behavior. The other options are factually or reasonably incorrect.

36) Choice A is the best answer because it directly reflects the findings of the study as described in the text. The research highlights that wolves spend more time in REM sleep compared to dogs, especially in older individuals, making this choice the most logical completion of the discussion.

Choice B: This is incorrect because the study does not mention aggression or behaviors related to sleep disturbances; its focus is on sleep stages and physiological differences between wolves and dogs.

Choice C: This option introduces an unsupported claim about stereotypic pacing behaviors before REM sleep, which is not mentioned in the study or findings.

Choice D: This is incorrect because the study explicitly mentions differences between wolves and dogs in terms of REM sleep, particularly as animals age. Thus, stating they have "identical" patterns contradicts the research findings.

37) Choice B is the best answer because it most logically completes the text's discussion of Dr. Marra and colleagues' investigation of the evolution and biological role of the syrinx. The text indicates that the team found several instances of the syrinx developing and not disappearing in the lineages of various bird species the team examined. Furthermore, the text states that species that possess a syrinx also tend to have more complex vocalizations—a feature that supports communication among birds. Taken together, these details strongly support the hypothesis that the syrinx has persisted in some species because it has a function that contributes to effective communication in those species. Choice A is incorrect because the text doesn't address any non-avian species. Choice C is incorrect because the text doesn't make predictions about the evolutionary future of the species Dr. Marra and colleagues examined, and although the implication of the text is that the syrinx likely does serve a function for the communication system, nothing in the text indicates that the syrinx will become more widespread in the future. Choice D is incorrect. Although the text does suggest an association between having a syrinx and more complex vocalizations, it doesn't claim that the syrinx causes the vocalizations to become more complex, nor does it address the relative complexity of the vocalizations at different periods of time.

38) Choice C is the best answer because it logically extends the passage's information: the presence of efficient gills and elevated hemoglobin levels in fish living near cold-water corals supports the hypothesis that these adaptations improve oxygen uptake in deep, oxygen-poor environments.

Choice A is incorrect because it incorrectly suggests that gills originated in land-dwelling animals, which contradicts evolutionary evidence—gills predate lungs in vertebrate history.

Choice B is incorrect because the passage links gills to oxygen intake, not temperature regulation.

Choice D is incorrect because no information is provided about coral spawning or fluctuations in hemoglobin during such events.

39) Option A is correct because it aligns with historical evidence about Inca society, which emphasized reciprocity (*ayni*) and collective well-being as guiding principles. The Incas evaluated morality based on how individual actions supported societal roles and contributed to communal harmony, rather than relying on universal moral rules independent of context.

40) Lesson: Correct punctuation ensures that nonessential phrases are set off properly without implying an independent clause.

Traps:

1. Overusing semicolons for phrases that aren't independent clauses.
2. Incorrectly substituting colons when the phrase does not introduce a new clause.
3. Misinterpreting a modifying phrase as a separate, standalone idea.
4. Overcomplicating punctuation, which can obscure the sentence's intended meaning.

Explanation:

Option A is the best answer because it uses commas to correctly set off the nonessential explanatory phrase "largely due to its size and the damage sustained" from the main clause ("The ship took less than three hours to sink despite being touted as unsinkable"). This punctuation choice clearly indicates that the phrase provides additional information.

Options B, C, and D are incorrect because they use semicolons or colons. Such punctuation is inappropriate here since the phrase following "sink" is not an independent clause but rather a modifying phrase that explains why the ship sank.

41) Lesson: Punctuation and Sentence Structure; Clarity in Providing Additional Information.

Trap: Unnecessary commas Incorrect use of conjunctions Unnecessary periods

Explanation:

Choice C is correct because "exclamation translated" effectively uses "translated" as a descriptor for the noun "exclamation" without introducing extraneous punctuation or conjunctions. This construction maintains the flow and clarity of the sentence.

Choice A is incorrect because the comma after "exclamation" unnecessarily separates the descriptor "translated," disrupting the sentence structure.

Choice B is incorrect because the conjunction "and" incorrectly suggests that "translated" is an additional, separate element rather than a modifier of "exclamation."

Choice D is incorrect because the period breaks the sentence into two parts, which is inappropriate for the intended sentence flow.

42)

Option B is correct because the semicolon after "Lagos" is conventionally used to distinguish the first item ("*The Joys*…Lagos") and the second item ("*A Kind*…Nigeria") in the series. Moreover, the comma after "*Marriage*" accurately separates the title *A Kind of Marriage* from the additional phrase ("a television…Nigeria") that describes it.

Option A is incorrect because the comma after "Lagos" doesn't align with the semicolon used later in the series to separate the second item from the third item.

Option C is incorrect for the same reason, and also because a colon can't be used in this way to separate the title *A Kind of Marriage* from its descriptive phrase ("a television…Nigeria").

Option D is incorrect because it fails to use appropriate punctuation to separate the title *A Kind of Marriage* from its descriptive phrase ("a television…Nigeria").

43) Option A is correct because the use of "would" here suggests a past future action from the perspective of 1994, indicating that the recognition of Wang Zhenyi's achievements by naming a crater on Venus was a significant event that was completed in the past. This choice aligns with the historical context of the sentence, reflecting that the recognition happened after Wang Zhenyi's death.

Option B is incorrect because the infinitive "to recognize" does not fit the context, as it suggests an incomplete action or future intention rather than a completed historical event.

Option C is incorrect because the perfect participle phrase "having finally recognized" implies that the recognition occurred before another action, which does not fit the sentence context. The sentence needs a form that reflects an event completed in the past without implying a sequence.

Option D is incorrect because, while this option uses the present participle "finally recognizing" to provide descriptive detail, it does not convey the past completion of the recognition as effectively as Option A. It might imply an ongoing action rather than a completed historical event.

44) Option B is correct because the subject "advocacy" is singular, requiring the singular verb "was." The sentence refers to a specific past event (1986), making "was" the appropriate choice.

Option A is incorrect because "are" is plural and does not align with the singular subject "advocacy."

Option C is incorrect because "have been" implies a continuous or ongoing action, which does not fit the context of a single historical event.

Option D is incorrect because "were" is plural and does not match the singular subject "advocacy."

45)

Option A is correct because the past perfect tense "had already served" is used to indicate that Misaka's service in World War II occurred before his subsequent achievements of winning championships and joining the Knicks. This tense establishes a clear sequence of events, making it grammatically and conceptually accurate.

Option B is incorrect because the present tense "serves" is inappropriate as the context refers to past events. It does not align with the historical timeline of Misaka's accomplishments.

Option C is incorrect because the past continuous tense "was already serving" implies an ongoing action during World War II, which does not clearly indicate that his service was completed before other events.

Option D is incorrect because, while the simple past tense "served" is grammatically correct, it fails to establish that Misaka's service occurred before his later achievements, making it less precise for expressing the intended sequence of events.

46) Choice D is the best answer. The singular possessive noun *quote's* correctly indicates that there is only one quote being referred to. The singular possessive determiner *its* agrees in number with the singular possessive noun *quote's,* emphasizing the idea that the simplicity of this single quote conceals deeper philosophical complexity.

Option A is incorrect because *quotes'* refers to multiple quotes, which does not match the context of discussing a single quote (*Cogito, ergo sum*).

Option B is incorrect because it uses *quotes* (plural noun) and *their* (plural possessive determiner), which do not align with the singular context.

Option C is incorrect because it uses *quote's* (singular possessive noun) but incorrectly pairs it with *their* (plural possessive determiner).

47) Choice D is the correct answer because "wasn't" maintains the past tense, which aligns with the rest of the sentence that describes Steinbeck's earlier career in journalism. The sentence discusses something from Steinbeck's past, so the past tense is needed to keep the sentence consistent.

Choice A is incorrect because "haven't been" is not only the wrong tense (present perfect) but also incorrect in terms of subject-verb agreement. The subject "prowess" is singular, so "haven't" is inappropriate.

Choice B is incorrect because "aren't" is present tense, which does not fit with the past-tense context of the sentence.

Choice C is incorrect because "isn't" is also in the present tense, which does not match the past tense narrative.

48) Lesson: Understanding the difference between possessive pronouns and contractions.

Traps: Misusing contractions (e.g., "it's" for "it is")

Using plural pronouns for a singular subject.

Choice C ("its") is correct because it is the possessive form referring to the aye-aye.

Choice A ("they're") is a contraction for "they are" and does not fit a singular subject.

Choice B ("their") is used for plural subjects.

Choice D ("it's") is a contraction for "it is," which is incorrect in this context.

49) Choice B is the best answer because the passage focuses on the contrast between the apparent ease of Patterson's writing and the reality of his thorough preparation. The passage states that although Patterson's style appears natural, this effect results from detailed planning and his consistent practice of treating each new novel as a fresh undertaking.

Choice A is incorrect because the passage does not discuss how Patterson selects genres.

Choice C is incorrect because it refers to stories Patterson feels "comfortable" encountering for the first time, which the passage does not mention—it says only that he treats each story *as if* he were seeing it for the first time.

Choice D is inaccurate because it suggests that Patterson is writing unfamiliar stories, whereas the passage clarifies that he is revisiting his own stories with a fresh mindset.

50) Choice B is the best answer because it most accurately describes the overall structure of the text. The text begins by explaining that human activities influence carbon and nitrogen levels in the ecosystem, but how deeply these effects are seen in the soil remains an unresolved question. Next, the text summarizes Dr. Harper and his team's hypothesis regarding this question—which is that the different effects on carbon and nitrogen levels associated with different types of habitat types would also be observed below the surface layer—and then briefly explains the methods they used to test this hypothesis. Finally, the text states that the researchers found that at depths below the surface layer, carbon and nitrogen decreased to similarly low levels across all habitat types. This conflicts with the team's hypothesis presented earlier in the text. Thus, the text introduces an unresolved scientific question, presents a research team's hypothesis about that question, and then describes an observation that the team made that conflicted with their hypothesis.

The other options are factually or reasonably incorrect.

51) Choice A is the correct answer.

Given:

Let $k = a + b + c$

Expression:

$x^2 - a^2 - b^2 - c^2 - 2ab - 2ac - 2bc + 2x(a + b + c)$

Step 1: Group like terms

Group the terms:

$x^2 + 2x(a + b + c) - \left(a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right)$

Step 2: Recognize the identities

$a + b + c = k$

$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = (a + b + c)^2 = k^2$

So we get:

$x^2 + 2xk - k^2$

Match with the given options

A. $x^2 + 2xk - k^2$ correct

B. $x^2 + 2kx - k^2 + abc$ (Extra term)

C. $(x+k)^2$ (Does not account for minus $2(ab+bc+ac)$)

D. $(x+a+b)^2+c^2$ (Not equivalent)

Final Simplified Expression:

$$\boxed{x^2+2xk-k^2}$$

52) Choice B is the correct answer.

We are given the following relationships:

1. $r=\sqrt[5]{k}$

2. $r^{15a-8}=k^3$

We are tasked with finding the value of a.

Step 1: Express r in terms of k

The first equation gives:

$$r=\sqrt[5]{k}$$

This can be rewritten as:

$$r=k^{1/5}$$

Step 2: Substitute $r=k^{1/5}$ into the second equation

Substitute $r=k^{1/5}$ into the second equation $r^{15a-8}=k^3$:

$$(k^{1/5})^{15a-8}=k^3$$

Step 3: Simplify the left-hand side

Using the power of a power rule, $(x^m)^n=x^{m\cdot n}$, we get:

$$k^{(1/5)\cdot(15a-8)}=k^3$$

This simplifies to:

$$k^{\frac{15a-8}{5}}=k^3$$

Step 4: Equate the exponents

Since the bases are the same, we can equate the exponents:

$$\frac{15a-8}{5}=3$$

Step 5: Solve for a

Multiply both sides of the equation by 5 to eliminate the denominator:

$$15a-8=15$$

Add 8 to both sides:

$$15a=23$$

Finally, divide both sides by 15:

$$a=\frac{23}{15}$$

Thus, the value of a is $\boxed{\frac{23}{15}}$.

53) Choice C is the correct answer.

Given the function:

$$f(x)=a^x-b$$

It passes through points (p,q) and $(2p,4q)$, and we know:

$$a^p=2q$$

We want to find b.

Step 1: Use the point (p,q)

Since $f(p)=q$,

$$f(p)=a^p-b=q$$

Rearranged:

$b = a^p - q$

Step 2: Use the point $(2p, 4q)$

Since $f(2p) = 4q$,

$f(2p) = a^{2p} - b = 4q$

Rewrite a^{2p} as $(a^p)^2$:

$(a^p)^2 - b = 4q$

Step 3: Substitute $a^p = 2q$ **and** $b = a^p - q$

Replace a^p with $2q$, and b with $a^p - q$:

$(2q)^2 - (2q - q) = 4q$

$4q^2 - (2q - q) = 4q$

$4q^2 - q = 4q$

Step 4: Simplify and solve for q

$4q^2 - q = 4q \implies 4q^2 - q - 4q = 0 \implies 4q^2 - 5q = 0$

Factor out q:

$q(4q - 5) = 0$

So,

$q = 0 \quad \text{or} \quad 4q - 5 = 0 \implies q = \frac{5}{4}$

Step 5: Find b

Since $q = 0$ would make $a^p = 0$, which is impossible (assuming $a > 0$), we take:

$q = \frac{5}{4}$

Recall:

$b = a^p - q = 2q - q = q = \frac{5}{4}$

Thus, the correct answer is $\boxed{\frac{5}{4}}$

54) Choice B is the correct answer.

Step 1: Evaluate $f(x)$ **at the two values**

Given:

$f(x) = A \cdot b^{\sqrt{x}}$

Evaluate at $x = 9$ and $x = 16$:

$f(9) = A \cdot b^{\sqrt{9}} = A \cdot b^3$

$f(16) = A \cdot b^{\sqrt{16}} = A \cdot b^4$

Step 2: Use the percent increase formula

The percent increase from $x = 9$ to $x = 16$ is:

$c = \left(\frac{f(16) - f(9)}{f(9)} \right) \cdot 100$

Substitute the values:

$c = \left(\frac{A \cdot b^4 - A \cdot b^3}{A \cdot b^3} \right) \cdot 100$

Step 3: Simplify the expression

Factor A out:

$c = \left(\frac{A(b^4 - b^3)}{Ab^3} \right) \cdot 100 = \left(\frac{b^4 - b^3}{b^3} \right) \cdot 100$

Now split the numerator:

$= \left(\frac{b^4}{b^3} - \frac{b^3}{b^3} \right) \cdot 100 = (b - 1) \cdot 100$

Alternatively, write:

$c = 100(b^{4-3} - 1)$

Thus, the correct answer is $\boxed{\text{Option B: } c = 100(b^{4-3} - 1)}$

55) Choice C is the correct answer.

Given:

- $f(x) = ab^x$
- $f(m-2) = 1.2f(m) + 0.64f(m-1)$

Step 1: Write each function in terms of a and b

$f(m-2) = ab^{m-2}, \quad f(m-1) = ab^{m-1}, \quad f(m) = ab^m$

Substitute into the equation:

$ab^{m-2} = 1.2ab^m + 0.64ab^{m-1}$

Step 2: Divide both sides by ab^{m-2}

Since $a > 0, b > 0$, and $b \neq 0$, we can divide safely:

$1 = 1.2b^2 + 0.64b$

Step 3: Rearrange into standard quadratic form

$1.2b^2 + 0.64b - 1 = 0$

Multiply by 100 to eliminate decimals:

$120b^2 + 64b - 100 = 0$

Divide all terms by 4:

$30b^2 + 16b - 25 = 0$

Step 4: Solve using the quadratic formula

The quadratic formula is:

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

For $30b^2 + 16b - 25 = 0$, we have:

- $A = 30$
- $B = 16$
- $C = -25$

$$b = \frac{-16 \pm \sqrt{16^2 - 4(30)(-25)}}{2(30)}$$

$$b = \frac{-16 \pm \sqrt{256 + 3000}}{60} = \frac{-16 \pm \sqrt{3256}}{60}$$

$\sqrt{3256} \approx 57.06$

$$b = \frac{-16 + 57.06}{60} \approx \frac{41.06}{60} \approx 0.6843$$

We discard the negative root since $b > 0$.

Thus, the correct answer is $\boxed{b \approx 0.684}$

56) Choice A is the correct answer.

We are given:

$\left(\frac{2}{5}x - \frac{37.5}{5}\right)^2 - 100 = 0$

Step 1: Factor out $\frac{2}{5}$ from inside the square

$\left(\frac{2}{5}(x - 18.75)\right)^2 - 100 = 0$

Step 2: Square the expression

$\left(\frac{2}{5}\right)^2(x - 18.75)^2 - 100 = 0$
$\Rightarrow \frac{4}{25}(x - 18.75)^2 - 100 = 0$

Step 3: Solve for $(x - 18.75)^2$

Add 100:

$\frac{4}{25}(x - 18.75)^2 = 100$

Multiply both sides by 25:

$4(x - 18.75)^2 = 2500$

Divide by 4:

$(x - 18.75)^2 = 625$

Now raise both sides to the third power:

$$(x - 18.75)^6 = 625^3$$

But note:

$$625 = 5^4 \Rightarrow (5^4)^3 = 5^{12}$$

Thus, the correct answer is $\boxed{5^{12}}$

57) Choice D is the correct answer.

Step 1: Understand the general form

Given:

- The function is of the form: $f(x) = A \cdot B^{x+C}$
- $f(3) = 640{,}000$
- We want to find which option shows $f(4) = k$ as either a coefficient or a base in the expression.

Step 2: Check each option to see which satisfies $f(3) = 640{,}000$

Option A: $f(x) = 2{,}500 \cdot 5^{x+1}$

$f(3) = 2{,}500 \cdot 5^4 = 2{,}500 \cdot 625 = 1{,}562{,}500 \neq 640{,}000$ Not correct

Option B: $f(x) = 2{,}500 \cdot 5^{x-1}$

$f(3) = 2{,}500 \cdot 5^2 = 2{,}500 \cdot 25 = 62{,}500$ Not correct

Option C: $f(x) = 2{,}500 \cdot 4^{x+2}$

$f(3) = 2{,}500 \cdot 4^5 = 2{,}500 \cdot 1024 = 2{,}560{,}000$ Not correct

Option D: $f(x) = 2{,}500 \cdot 4^{x+1}$

$f(3) = 2{,}500 \cdot 4^4 = 2{,}500 \cdot 256 = 640{,}000$ Matches the given

Step 3: Compute $f(4)$**using Option D**

$f(4) = 2{,}500 \cdot 4^{4+1} = 2{,}500 \cdot 4^5 = 2{,}500 \cdot 1024 = 2{,}560{,}000$
$\Rightarrow k = 2{,}560{,}000$

Thus, the function: $f(x) = 2{,}500 \cdot (4)^{x+1}$

- Satisfies $f(3) = 640{,}000$
- And shows $f(4) = 2{,}560{,}000$ (value of k) in the expression as the result of a base raised to a power.

Thus, the correct answer is $\boxed{f(x) = 2{,}500 \cdot (4)^{x+1}, \quad k = 2{,}560{,}000}$

58) Choice C is the correct answer.

Step 1: Understand the given function

We are given:

$$V(t) = 2000(1.10)^{\frac{t-2005}{3}}$$

- $V(t)$ is the value of the sculpture in dollars at year t.
- The base is 1.10, which indicates a 10% increase.
- The exponent is $\frac{t-2005}{3}$, which affects how often the increase happens.

Step 2: Recall the form of exponential growth

The general form of an exponential growth function is:

$$V(t) = V_0 \cdot (1 + r)^{\frac{t-t_0}{n}}$$

Where:

- V_0 is the initial value,
- r is the growth rate per interval,
- n is the number of years per growth interval (what we're trying to find),
- t_0 is the starting year.

Step 3: Compare with the given function

Compare:

$$V(t) = 2000(1.10)^{\frac{t-2005}{3}}$$

with:

$$V(t) = V_0 \cdot (1 + r)^{\frac{t-t_0}{n}}$$

From this comparison:

- $V_0 = 2000$

- $1 + r = 1.10 \rightarrow$ so $r = 0.10$, or 10%
- $t_0 = 2005$
- $\frac{t-2005}{3}$ tells us that the increase happens every 3 years

Step 4: Conclude

Since the power is $\frac{t-2005}{3}$, the value increases by 10% every 3 years.

Thus, the correct answer is $\boxed{n = 3}$

59) The correct answer is option B.

This question asks which function $g(x)$ matches the recursive relationship $g(x + 2) = \frac{1}{9}g(x)$.

Given Information:

- The equation $g(x + 2) = \frac{1}{9}g(x)$ describes how $g(x)$ changes as x increases by 2.

Steps to Solve:

Step 1:

Recognize that $\frac{1}{9}$ is equivalent to $\left(\frac{1}{3}\right)^2$.

Step 2:

The function must follow the pattern $g(x) = A\left(\frac{1}{3}\right)^x$ where the function decreases by a factor of $\frac{1}{9}$ for every increase of 2 in x.

Step 3:

Option B $27\left(\frac{1}{3}\right)^x$ matches this form with the correct base and factor.

Why the Other Options Are Incorrect:

- **Option A** $9\left(\frac{1}{9}\right)^x$**:** Incorrect because it gives a factor of $\frac{1}{9}$ for a change of 1 in x.
- **Option C** $\frac{1}{3}(3)^x$**:** Incorrect because it increases instead of decreases.
- **Option D** $\left(\frac{1}{9}\right)^x$**:** Incorrect because it uses the wrong base and doesn't match the transformation.

Thus, the correct option is B.

60) Choice C is the correct answer.

Given:

$$\sqrt{579 + 48\sqrt{3}} = r + \sqrt{p}, \text{ where } r, p \in \mathbb{Z}^+$$

We are to find the value of $r + p$.

Step 1: Let the expression equal $r + \sqrt{p}$

Assume:

$$\sqrt{579 + 48\sqrt{3}} = r + \sqrt{p}$$

Square both sides:

$$579 + 48\sqrt{3} = (r + \sqrt{p})^2 = r^2 + 2r\sqrt{p} + p$$

Step 2: Match rational and irrational parts

From:

$$579 + 48\sqrt{3} = r^2 + p + 2r\sqrt{p}$$

Equating parts:

Rational part:

$$r^2 + p = 579 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Irrational Part

$$2r\sqrt{p} = 48\sqrt{3} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Step 3: Solve the irrational part equation

From (2) :

$$2r\sqrt{p} = 48\sqrt{3}$$

$$\Rightarrow r\sqrt{p} = 24\sqrt{3}$$

Now divide both sides:

$$\frac{r\sqrt{p}}{\sqrt{3}} = 24$$

$\Rightarrow r \cdot \sqrt{\frac{p}{3}} = 24$

Now square both sides:

$r^2 \cdot \frac{p}{3} = 576$

$\Rightarrow r^2 p = 1728 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

Step 4: Use substitution from Equation (1)

From equation (1):

$p = 579 - r^2$

Substitute into equation (3):

$r^2(579 - r^2) = 1728$

$\Rightarrow 579r^2 - r^4 = 1728$

$\Rightarrow r^4 - 579r^2 + 1728 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$

Step 5: Solve the quadratic in terms of $x = r^2$

Let x= r^2.

Then equation (4) becomes:

$x^2 - 579x + 1728 = 0$

Use the quadratic formula:

$x = \frac{579 \pm \sqrt{579^2 - 4 \cdot 1728}}{2}$

Calculate discriminant:

$\Delta = 579^2 - 6912 = 335241 - 6912 = 328329$

Checkthe square root:

$\sqrt{328329} = 573 \quad (\text{since } 573^2 = 328329)$

Now solve:

$x = \frac{579 \pm 573}{2}$

Take the positive root:

$x = \frac{579+573}{2} = \frac{1152}{2} = 576$

$\Rightarrow r^2 = 576 \Rightarrow r = \sqrt{576} = 24$

Step 6: Find the value of p

Substitute $r = 24$ into equation (1):

$r^2 + p = 579 \Rightarrow 576 + p = 579 \Rightarrow p = 3$

Step 7: Calculate $r + p$

$r + p = 24 + 3 = \boxed{27}$

61) Choice A is the correct answer.

We are given the equation:

$(x + 2)^2 + (x^2 + 4x) = (x + 2)(x - 3)$

and the condition $x \neq -2$.

Let's follow the below steps:

Step 1: Expand both sides

Left-hand side (LHS):

$(x + 2)^2 + (x^2 + 4x) = x^2 + 4x + 4 + x^2 + 4x = 2x^2 + 8x + 4$

Right-hand side (RHS):

$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

Step 2: Set both sides equal

$2x^2 + 8x + 4 = x^2 - x - 6$

Step 3: Bring all terms to one side

$2x^2 + 8x + 4 - x^2 + x + 6 = 0$

$x^2 + 9x + 10 = 0$

Step 4: Solve the quadratic

$x^2 + 9x + 10 = 0$

Use the Quadratic Formula

The general form is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $x^2 + 9x + 10 = 0$, we have:

- $a = 1$
- $b = 9$
- $c = 10$

Now plug into the formula:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(10)}}{2(1)} = \frac{-9 \pm \sqrt{81 - 40}}{2}$$

$$x = \frac{-9 \pm \sqrt{41}}{2}$$

Step 5: Apply the condition $x \neq -2$

Neither solution is -2, so the condition $x \neq -2$ is automatically satisfied.

Thus, the correct answer is $\boxed{\frac{-9 \pm \sqrt{41}}{2}}$

62) Choice C is the correct answer.

To find the value of m where the revenue line does not intersect the cost curve, we follow the steps similar to the previous explanation.

The given equations are:

- **Revenue equation:** $y = -1.5x + m$
- **Cost equation:** $y = 0.5x^2 + 2x - m$

Let's follow the steps below:

Step 1: Set the two equations equal to each other

$-1.5x + m = 0.5x^2 + 2x - m$

Step 2: Rearrange the equation

$0.5x^2 + 2x - m + 1.5x - m = 0$

$0.5x^2 + 3.5x - 2m = 0$

Step 3: Analyze the discriminant

The discriminant Δ of the quadratic equation $0.5x^2 + 3.5x - 2m = 0$ is:

$\Delta = (3.5)^2 - 4(0.5)(-2m)$

$\Delta = 12.25 + 4m$

Step 4: Condition for no intersection

For the line and the curve not to intersect, the discriminant must be negative, i.e., $\Delta < 0$:

$12.25 + 4m < 0$

$4m < -12.25$

$m < -3.0625$

Conclusion

For the line and the curve not to intersect, m must be less than -3.0625.

From the provided options, -5.5 is the only value that satisfies this condition.

Thus, the correct answer is $\boxed{-5.5}$

63) Choice B is the correct answer.

We need to determine the time t (in years) after 1980 such that the number of transistors, $N(t)$, exceeds $8,000,000$.

Let's solve this step-by-step:

Step 1: Write the doubling formula

The number of transistors at time t is modeled as:

$$N(t) = 125,000 \cdot 2^{\frac{t}{1.5}}$$

where t is the number of years since 1980. We want:

$$N(t) > 8,000,000$$

Step 2: Set up the inequality

$$125,000 \cdot 2^{\frac{t}{1.5}} > 8,000,000$$

Step 3: Divide both sides by $125,000$

$$2^{\frac{t}{1.5}} > \frac{8,000,000}{125,000}$$

Simplify the right-hand side:

$$2^{\frac{t}{1.5}} > 64$$

Step 4: Take the logarithm (base 2) of both sides

$$\frac{t}{1.5} > \log_2(64)$$

Since $\log_2(64) = 6$, we get:

$$\frac{t}{1.5} > 6$$

Step 5: Solve for t

$$t > 6 \cdot 1.5$$

$$t > 9$$

Thus, the number of transistors exceeds $8,000,000$ 9 years after 1980. The corresponding year is:

$$1980 + 9 = 1989$$

Step 6: Check the exact doubling cycle

To determine when the condition is first exceeded:

At $t = 9$ (1989):

$$N(9) = 125,000 \cdot 2^{\frac{9}{1.5}}$$

$$= 125,000 \cdot 2^6 = 125,000 \cdot 64 = 8,000,000$$

So it is just equal to $8,000,000$.

At $t = 10.5$ (1991):

$$N(10.5) = 125,000 \cdot 2^{\frac{10.5}{1.5}} = 125,000 \cdot 2^7 = 125,000 \cdot 128 = 16,000,000$$

The calculation shows that Moore's law predicts the number of transistors to double every 1.5 years. Starting from $125,000$ in 1980, the transistors reached $8,000,000$ exactly in 1989 but only exceeded that number in 1991.

Thus, the transistors exceeded $8,000,000$ in the year 1991.

64) Choice D is the correct answer.

Given:

$$6x^4 + 41x^2 + 70 = (3x^2 + a)(2x^2 + b) = (3x^2 + c)(2x^2 + d)$$

where

a, b are positive integers

c, d are positive non-integers

Step 1: Write down the system from expansion

Expand $(3x^2 + A)(2x^2 + B)$:

$$= 6x^4 + (3B + 2A)x^2 + AB$$

Matching coefficients with $6x^4 + 41x^2 + 70$, get:

$$3B + 2A = 41$$

$$AB = 70$$

Step 2: Express B from the second equation

From

$AB = 70 \implies B = \frac{70}{A}$

Substitute into the first equation:

$3 \times \frac{70}{A} + 2A = 41$

Multiply both sides by A to clear the denominator:

$3 \times 70 + 2A^2 = 41A$

Simplify:

$210 + 2A^2 = 41A$

Rearranged as:

$2A^2 - 41A + 210 = 0$

Step 3: Solve the quadratic for A

Use the quadratic formula:

$$A = \frac{41 \pm \sqrt{41^2 - 4 \times 2 \times 210}}{2 \times 2}$$

Calculate discriminant:

$41^2 = 1681, \quad 4 \times 2 \times 210 = 1680$

$\Delta = 1681 - 1680 = 1$

So,

$A = \frac{41 \pm 1}{4}$

Two solutions:

$A_1 = \frac{42}{4} = 10.5, \quad A_2 = \frac{40}{4} = 10$

Step 4: Interpret results

Since a, b are positive integers, $A = a$ must be an integer, so

$a = 10$

For this a,

$b = \frac{70}{10} = 7$

Step 5: Find c for the non-integer factorization

The other factorization has c, d positive non-integers.

So c corresponds to the other root:

$c = 10.5 = \frac{21}{2}$

Step 6: Find $a + c$

$a + c = 10 + 10.5 = 20.5 = \frac{41}{2}$

Thus, the correct answer is $\boxed{\frac{41}{2}}$

65) Choice D is the correct answer.

Given:

- $x + y = \sqrt{a + 3}$
- $xy = a - 4$
- $x^2 + y^2 = a + 1$

We are to find:

$x^4 + y^4 \quad$ in terms of a

Step 1: Use identity for $x^2 + y^2$

We know:

$x^2 + y^2 = (x + y)^2 - 2xy$

Substitute the known values:

$a + 1 = (\sqrt{a + 3})^2 - 2(a - 4)$

$a + 1 = (a + 3) - 2a + 8$

$a+1=a+11-2a$
$\Rightarrow a+1=-a+11$

Check if both sides match:

$a+a=11-1=10 \Rightarrow 2a=10 \Rightarrow a=5$

So, we now know that:

$a=5$

Step 2: Compute x^4+y^4

We use the identity:

$x^4+y^4=(x^2+y^2)^2-2x^2y^2$

We already know:

- $x^2+y^2=a+1=6$
- $xy=a-4=1 \Rightarrow x^2y^2=(xy)^2=1^2=1$

Now compute:

$x^4+y^4=(6)^2-2(1)=36-2=34$

Step 3: Plug $a=5$ **into all the answer options**

Let's test which one gives 34:

Option A: $-a^2+16a-31$

$=-25+80-31=24$

Option B: $-a^2+20a-29$

$=-25+100-29=46$

Option C: $-a^2+18a-28$

$=-25+90-28=37$

Option D: $-a^2+18a-31$

$=-25+90-31=34$

This matches our earlier result.

Thus, the correct answer is $\boxed{-a^2+18a-31}$

66) Choice D is the correct answer.

Let's solve this step by step.

Step 1: Factor Theorem

According to the factor theorem, if $k-x$is a factor of the polynomial $P(x)$, then substituting $x=k$into $P(x)$ will result in $P(k)=0$.

The given polynomial is:

$P(x)=-x^2+\frac{1}{p}nk^2$

We are tasked to find the value of n when $p=29$ and $P(x)$ has $k-x$ as a factor.

Step 2: Substitute $x=k$ into $P(x)$

Substituting $x=k$ into $P(x)$:

$P(k)=-k^2+\frac{1}{p}nk^2$

Since $k-x$ is a factor, we know $P(k)=0$.

Thus:

$-k^2+\frac{1}{p}nk^2=0$

Step 3: Solve for n

Factor out k^2(since $k>0$):

$k^2\left(-1+\frac{1}{p}n\right)=0$

Since $k^2\neq 0$, the term in parentheses must equal 0:

$-1+\frac{1}{p}n=0$

Rearrange to solve for n:

$\frac{1}{p}n=1$

Multiply through by p:

$n = p$

Step 4: Substitute $p = 29$

When $p = 29$, we have:

$n = 29$

Step 5: Match the correct option

The correct answer is $n = 29$.

Incorrect Options

- **Option A (-29):** Incorrect because it reverses the positive value of n.
- **Option B ($-\frac{1}{29}$):** Incorrect because it incorrectly assumes a division relationship instead of multiplication.
- **Option C ($\frac{1}{29}$):** Incorrect because it inverts the scaling factor.

Thus, the correct answer is $n = 29$.

67) Choice A is the correct answer.

Given:

- We are given a projectile launched from a cliff 20 meters above the ground, and its height at time t is modeled by the quadratic function:

  $h(t) = at^2 + bt + 20$

- The projectile reaches its maximum height of 92 meters at time $t = 3$.
- We need to find the height of the projectile at time $t = 6$.

Step 1: Use the information about the maximum height

The maximum height occurs at the vertex of the parabola, and for a quadratic function $h(t) = at^2 + bt + c$, the vertex occurs at time $t = -\frac{b}{2a}$.

We are told that the maximum height occurs at $t = 3$, so:

$-\frac{b}{2a} = 3$

Solving for b:

$b = -6a$

Step 2: Use the maximum height of 92 meters

At time $t = 3$, the height of the projectile is 92 meters.

We substitute $t = 3$ into the height equation:

$h(3) = a(3)^2 + b(3) + 20 = 92$

Substitute $b = -6a$ into the equation:

$a(9) + (-6a)(3) + 20 = 92$

Simplify:

$9a - 18a + 20 = 92$

$-9a + 20 = 92$

Solve for a:

$-9a = 92 - 20$

$-9a = 72$

$a = -8$

Step 3: Find b

Now that we know $a = -8$, substitute it into $b = -6a$:

$b = -6(-8) = 48$

Step 4: Write the height equation

Now we have the complete equation for the height of the projectile:

$h(t) = -8t^2 + 48t + 20$

Step 5: Find the height at time $t = 6$

Substitute $t = 6$ into the equation:

$h(6) = -8(6)^2 + 48(6) + 20$

Simplify:

$h(6) = -8(36) + 288 + 20$

$h(6) = -288 + 288 + 20$

$h(6) = 20$

Thus, the height of the projectile at time $t = 6$ is $\boxed{20}$ meters.

68) Choice B is the correct answer.

To determine the minimum value of the quadratic function $f(x) = x^2 - 84x + 4,356$, follow these steps:

Step 1: Find the value of x at the vertex

The minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at:

$x = -\frac{b}{2a}$

Here, $a = 1$ and $b = -84$.

Substituting these values:

$x = -\frac{(-84)}{(2)\cdot(1)} = \frac{84}{2} = 42$

Thus, the minimum value occurs when $x = 42$.

Step 2: Calculate $f(42)$

Substitute $x = 42$ into the function:

$f(42) = (42)^2 - 84 \cdot 42 + 4,356$

Evaluate each term:

- $42^2 = 1,764$,
- $-84 \cdot 42 = -3,528$,
- Constant term $= 4,356$.

Adding these:

$f(42) = 1,764 - 3,528 + 4,356 = 2,592$

Thus, the minimum revenue is $\$2,592$.

Step 3: Interpret the result

The rewritten function confirms that the minimum revenue occurs at $x = 42$ (representing 42 hundreds of products or $4,200$ individual products sold).

The minimum value is $\$2,592$.

Incorrect Options:

- Option A ($\$2,220$): Incorrect. This value does not match the correct calculation from substituting $x = 42$.
- Option C ($\$2,400$): Incorrect. This is not the true minimum value.
- Option D ($\$2,800$): Incorrect. This does not match the minimum value calculated for $x = 42$.

Final Answer:

The correct answer is: $\boxed{\text{Minimum revenue is } \$2,592 \textit{ at } 4200 \text{ products sold}}$

69) Choice D is the correct answer.

We are given the quadratic in the form:

$mx^2 + (m+n)x + \frac{mn}{2} = 0$

Let's apply the quadratic root formulas.

Step 1: Use the sum of roots formula

The sum of roots is:

$-\frac{b}{a} = -\frac{m+n}{m}$

We are told:

$-\frac{m+n}{m} = -\frac{3}{2}$
$\Rightarrow \frac{m+n}{m} = \frac{3}{2}$
$\Rightarrow 1 + \frac{n}{m} = \frac{3}{2}$
$\Rightarrow \frac{n}{m} = \frac{1}{2}$
$\Rightarrow \frac{m}{n} = 2$

Step 2: Use the product of roots formula

The product of roots is:

$\frac{c}{a} = \frac{\frac{mn}{2}}{m} = \frac{n}{2}$

We are told:

$\frac{n}{2} = \frac{1}{2} \Rightarrow n = 1$
$\Rightarrow m = 2 \Rightarrow \frac{m}{n} = \frac{2}{1}$

Therefore, the correct value is $\boxed{2}$.

70) Choice D is the correct answer.

We are given the quadratic expression:

$mx^2 + 120x + n$

where m and n are positive constants.

Additionally, it is stated that $x + p$ is a factor of the quadratic expression, where p is a positive constant.

Step 1: Factorization of the quadratic expression

Since $x + p$ is a factor of the quadratic expression, we can express the quadratic as:

$mx^2 + 120x + n = (x + p)(mx + q)$

where q is some constant.

Step 2: Expand the factorized form

Expanding $(x + p)(mx + q)$ gives:

$(x + p)(mx + q) = mx^2 + qx + mpx + pq$

Simplifying, we get:

$mx^2 + (mp + q)x + pq$

Step 3: Compare coefficients

Now, compare the coefficients of like powers of x from both the expanded form and the original quadratic expression.

- For the x^2 term: $m = m$ (which is trivially true).
- For the x term: $mp + q = 120$.
- For the constant term: $pq = n$.

Step 4: Solve for m and n in terms of p

From the equation $mp + q = 120$, we can solve for q:

$q = 120 - mp$

Substitute this into the constant term equation $pq = n$:

$p(120 - mp) = n$

Expanding:

$120p - mp^2 = n$

Thus, the relationship between m, n, and p is:

$n = 120p - mp^2$

Step 5: Maximize mn

We want to maximize mn.

Substituting $n = 120p - mp^2$ into the expression for mn:

$mn = m(120p - mp^2)$

Expanding:

$mn = 120mp - m^2p^2$

Now, we maximize this expression with respect to p.

Step 6: Take the derivative with respect to p

To find the maximum, we take the derivative of mn with respect to p and set it equal to zero:

$\frac{d}{dp}(120mp - m^2p^2) = 120m - 2m^2p = 0$

Solving for p:

$120m = 2m^2p$

$p = \frac{120}{2m} = \frac{60}{m}$

Step 7: Substitute $p = \frac{60}{m}$ into the expression for n

Substitute $p = \frac{60}{m}$ into the equation $n = 120p - mp^2$:

$$n = 120 \times \frac{60}{m} - m \times \left(\frac{60}{m}\right)^2$$

Simplifying:

$$n = \frac{7200}{m} - \frac{3600}{m}$$

$$n = \frac{3600}{m}$$

Step 8: Calculate mn

Now, we compute mn:

$$mn = m \times \frac{3600}{m} = 3600$$

Thus, the greatest possible value of mn is: $\boxed{3600}$

71) Choice C is the correct answer.

We are given a rational function:

$$h(x) = \frac{p(x)}{2x+1} = \frac{ax+b}{2x+1}$$

We are told that $p(x) = ax + b$ is linear and the graph of $h(x)$ passes through the following points:

x	$h(x)$
-2.5	2
0.5	-2
3.5	0

Step 1: Use the given points to form equations

We know:

$$h(x) = \frac{ax+b}{2x+1}$$

Using the point $(x = -2.5, h(x) = 2)$:

$$2 = \frac{a(-2.5)+b}{2(-2.5)+1} = \frac{-2.5a+b}{-5+1} = \frac{-2.5a+b}{-4}$$

Multiply both sides by -4 :

$$-8 = -2.5a + b \quad (1)$$

Using the point $(x = 0.5, h(x) = -2)$:

$$-2 = \frac{a(0.5)+b}{2(0.5)+1} = \frac{0.5a+b}{2}$$

Multiply both sides by 2:

$$-4 = 0.5a + b \quad (2)$$

Step 2: Solve the system of equations

From the equation (1):

$$-8 = -2.5a + b$$

From the equation (2):

$$-4 = 0.5a + b$$

Subtract equation (2) from equation (1):

$$(-8) - (-4) = (-2.5a + b) - (0.5a + b)$$

$$-4 = -3a$$

$$\Rightarrow a = \frac{4}{3}$$

Substitute $a = \frac{4}{3}$ into equation (2):

$$-4 = 0.5 \cdot \frac{4}{3} + b = \frac{2}{3} + b$$

$$\Rightarrow b = -4 - \frac{2}{3} = -\frac{12}{3} - \frac{2}{3} = -\frac{14}{3}$$

Step 3: Find the $y-$coordinate of the $y-$intercept of $p(x) = ax + b$

The $y-$intercept occurs at $x = 0$, so:

$p(0) = a \cdot 0 + b = b = -\frac{14}{3}$

Thus, the correct answer is $\boxed{-\frac{14}{3}}$

72) Choice C is the correct answer.

We are given the following system of equations:

$(1) \quad y = -2x^2 + 8x - k$
$(2) \quad x + y = k$

We are to find the largest integer greater than or equal to the value of k such that the system has exactly two distinct real solutions.

Step 1: Substitute the equation (1) into the equation (2)

From (2):

$x + y = k$

Substitute y from (1):

$x + (-2x^2 + 8x - k) = k$

Simplify:

$-2x^2 + 9x - k = k$

$-2x^2 + 9x - 2k = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

This is a quadratic in x.

For the system to have exactly two distinct real solutions, this quadratic must have two distinct real roots, which occurs when the discriminant is positive.

Step 2: Use the discriminant

General quadratic:

$ax^2 + bx + c = 0 \quad \Rightarrow \quad \text{Discriminant } D = b^2 - 4ac$

For the equation (3):

$a = -2, \quad b = 9, \quad c = -2k$

Then:

$D = 9^2 - 4(-2)(-2k) = 81 - 16k$

We require exactly two distinct real roots, so:

$D > 0 \Rightarrow 81 - 16k > 0 \Rightarrow k < \frac{81}{16}$

$\frac{81}{16} = 5.0625$

So, $k < 5.0625$

Step 3: Find the largest integer $\geq k$

We want the largest integer greater than or equal to the value of k for which the system has exactly two real solutions.

So, $k < 5.0625$, so the largest integer ≤ this is 5

Thus, the correct answer is $\boxed{5}$

73) Choice A is the correct answer.

Step 1: Substitute $y = x + 5$ into $y = x^2 + 10x + 24$

Start with the given system of equations:

$y = x + 5 \quad \text{and} \quad y = x^2 + 10x + 24$

Substitute $y = x + 5$ into the second equation:

$x + 5 = x^2 + 10x + 24$

Step 2: Rearrange into standard quadratic form

Rearrange the terms:

$0 = x^2 + 10x + 24 - x - 5$

Simplify:

$0 = x^2 + 9x + 19$

Step 3: Solve the quadratic equation

The quadratic equation is:

$$x^2 + 9x + 19 = 0$$

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1$, $b = 9$, and $c = 19$.

Substitute the values:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(19)}}{2(1)}$$

Simplify the discriminant:

$$x = \frac{-9 \pm \sqrt{81 - 76}}{2}$$

$$x = \frac{-9 \pm \sqrt{5}}{2}$$

Thus, the two solutions are:

$$x = \frac{-9 + \sqrt{5}}{2} \quad \text{and} \quad x = \frac{-9 - \sqrt{5}}{2}$$

Step 4: Approximate the solutions

For the greater solution:

$$x = \frac{-9 + \sqrt{5}}{2}$$

Approximating:

$$x \approx \frac{-9 + 2.236}{2} = \frac{-6.764}{2} \approx -3.38$$

For the smaller solution:

$$x = \frac{-9 - \sqrt{5}}{2}$$

Approximating:

$$x \approx \frac{-9 - 2.236}{2} = \frac{-11.236}{2} \approx -5.62$$

Step 5: Identify the greatest possible value of x

The greatest possible value of x is approximately:

$$\boxed{-3.38}$$

Incorrect Options:

- **Option** -2**:**This arises from an incorrect application of the quadratic formula, likely due to miscalculating the discriminant.
- **Option** -1**:** This is incorrect because it does not satisfy the quadratic equation and may result from ignoring the negative terms.
- **Option** 0**:** This is far from the range of solutions and may arise from assuming non-quadratic behavior of the equation.

Thus, the correct answer is A.

74) The correct answer is A.

Step 1: Analyze the given equations

1. The first equation, $y = 20$, represents a horizontal line at $y = 20$.

2. The second equation, $y = -2(x - 15)^2 + 20$, represents a downward-facing parabola with its vertex at $(15, 20)$.

The vertex form of the parabola shows that its maximum point (the vertex) is at $(15, 20)$.

Below this point, the parabola decreases symmetrically on either side.

Step 2: Check for intersections

To find the intersection points, set the equations equal to each other:

$$20 = -2(x - 15)^2 + 20$$

Simplify:

$$20 - 20 = -2(x - 15)^2$$

$$0 = -2(x - 15)^2$$

Divide by -2:

$$(x - 15)^2 = 0$$

Take the square root of both sides:

$x - 15 = 0$

$x = 15$

At $x = 15$, substituting into $y = 20$ confirms that the intersection occurs at the vertex of the parabola: $(15, 20)$.

Step 3: Verify the nature of the intersection

The parabola touches the horizontal line $y = 20$ at exactly one point—the vertex. Since the parabola opens downward, it does not cross or intersect the line at any other points.

Therefore, there is exactly one intersection point..

Incorrect Options

- **Option A ("Exactly one"):** Correct because the parabola and the line touch at one point only—the vertex.
- **Option B ("Exactly two"):** Incorrect because the parabola opens downward, and there are no two intersections.
- **Option C ("Infinitely many"):** Incorrect because the two graphs do not overlap.
- **Option D ("Zero"):** Incorrect because the graphs do intersect, but at exactly one point.

Thus, the graphs intersect at exactly one point, $(15, 20)$

75) Choice D is the correct answer.

We are given a problem involving five consecutive odd integers arranged in increasing order, with the first expressed as x.

We are tasked with finding the greatest possible value of x under the condition that the quotient of the product of the first and fifth integers and the sum of the second and third integers must be at most 12.

Step 1: Define the integers

The five consecutive odd integers are:

$x,\ x + 2,\ x + 4,\ x + 6,\ x + 8$

Step 2: Translate the condition into an inequality

The condition is that the quotient of the product of the first and fifth integers and the sum of the second and third integers must be at most 12.

We express this mathematically:

- First integer: x
- Fifth integer: $x + 8$
- Product: $x(x + 8)$

- Second integer: $x + 2$
- Third integer: $x + 4$
- Sum: $(x + 2) + (x + 4) = 2x + 6$

The condition becomes:

$\frac{x(x+8)}{2x+6} \leq 12$

Step 3: Solve the inequality

We begin by simplifying the inequality:

$\frac{x(x+8)}{2x+6} \leq 12$

Multiply both sides by $2x + 6$ (which is positive because x is positive):

$x(x + 8) \leq 12(2x + 6)$

Expand both sides:

$x^2 + 8x \leq 24x + 72$

Move all terms to one side:

$x^2 + 8x - 24x - 72 \leq 0$

Simplify:

$x^2 - 16x - 72 \leq 0$

This is a quadratic inequality.

To solve it, we first find the roots of the corresponding quadratic equation:

$x^2 - 16x - 72 = 0$

Using the quadratic formula:

$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(-72)}}{2(1)}$

$x = \frac{16 \pm \sqrt{256 + 288}}{2}$

$x = \frac{16 \pm \sqrt{544}}{2}$

$x = \frac{16 \pm 23.32}{2}$

So the two roots are approximately:

$x = \frac{16+23.32}{2} \approx 19.66$

$x = \frac{16-23.32}{2} \approx -3.66$

Since x must be a positive integer, we take the larger root:

$x \approx 19.66$

The greatest integer x that satisfies the inequality is $x = 19$.

Step 4: Verify the solution

Let's verify by substituting $x = 19$ into the inequality.

The integers are:

$19, 21, 23, 25, 27$

Product of the first and fifth integers: $19 \times 27 = 513$

Sum of the second and third integers: $21 + 23 = 44$

Now compute the quotient:

$\frac{513}{44} \approx 11.64$

Since $11.64 \leq 12$, the value $x = 19$ satisfies the condition.

Thus, the greatest possible value of x is: $\boxed{19}$

76) Choice B is the correct answer.

Step 1: Find the slope of the line m

Given line m:

$\frac{5}{2}x + \frac{3}{4}y = 8$

Rewrite in slope-intercept form $y = mx + c$:

$\frac{3}{4}y = 8 - \frac{5}{2}x$

Multiply both sides by $\frac{4}{3}$:

$y = \frac{4}{3} \times 8 - \frac{4}{3} \times \frac{5}{2}x = \frac{32}{3} - \frac{20}{6}x = \frac{32}{3} - \frac{10}{3}x$

So slope of m is

$m_m = -\frac{10}{3}.$

Step 2: Find the slope of line ℓ (perpendicular to m)

Slopes of perpendicular lines satisfy:

$m_\ell = -\frac{1}{m_m} = -\frac{1}{-\frac{10}{3}} = \frac{3}{10}.$

Step 3: Write equation of line ℓ passing through $\left(\frac{5}{3}, -\frac{4}{3}\right)$ with slope $\frac{3}{10}$

Use point-slope form:

$y - y_1 = m(x - x_1)$

$y - \left(-\frac{4}{3}\right) = \frac{3}{10}\left(x - \frac{5}{3}\right)$

$y + \frac{4}{3} = \frac{3}{10}x - \frac{3}{10} \times \frac{5}{3}$

$y + \frac{4}{3} = \frac{3}{10}x - \frac{15}{30} = \frac{3}{10}x - \frac{1}{2}$

Step 4: Find b, the y-intercept of ℓ (where $x = 0$)

Substitute $x = 0$:

$y + \frac{4}{3} = \frac{3}{10} \times 0 - \frac{1}{2} = -\frac{1}{2}$

$y = -\frac{1}{2} - \frac{4}{3} = -\frac{1}{2} - \frac{4}{3}$

Find the common denominator (6):

$y = -\frac{3}{6} - \frac{8}{6} = -\frac{11}{6}$

Thus, the value of b is $\boxed{-\frac{11}{6}}$

77) Choice D is the correct answer.

Let's follow the below steps:

Step 1: Let's expand the expression $(hx + k)(x + j)$:

$hx(x + j) + k(x + j) = hx^2 + hjx + kx + kj$

$= hx^2 + (hj + k)x + kj$

Now match this with the original expression:

$4x^2 + bx - 45$

So:

hx^2 matches with $4x^2 \rightarrow h = 4$

bx matches with $(hj + k)x \rightarrow b = hj + k$

Constant term $kj = -45$

Now we focus on the constant term:

$kj = -45 \Rightarrow k \cdot j = -45$

We are asked which of the four expressions must be an integer.

Let's test each:

Option A: $\frac{b}{h}$

We know $b = hj + k$, so $\frac{b}{h} = \frac{hj+k}{h}$ is not guaranteed to be an integer unless k is divisible by h, which is not always the case.

Option B: $\frac{b}{j}$

$\frac{b}{j} = \frac{hj+k}{j}$ – same issue, this depends on the specific values of h and k, so not guaranteed to be an integer.

Option C: $\frac{45}{h}$

We know nothing that guarantees h divides 45. For example, $h = 4$, and $\frac{45}{4}$ is not an integer.

Option D: $\frac{45}{j}$

Since $k \cdot j = -45$ and both k and j are integers, j must be a factor of 45. Therefore, $\frac{45}{j}$ will always be an integer.

So the correct answer is $\frac{45}{j}$.

78) Choice C is the correct answer.

Step 1: Given the line ℓ_1 has equation

$y = \frac{2}{3}x + b,$

so its slope is

$m_1 = \frac{2}{3}.$

Step 2: The line ℓ_2 is perpendicular to ℓ_1, so its slope m_2 is the negative reciprocal:

$m_2 = -\frac{1}{m_1} = -\frac{3}{2}.$

Step 3: Since ℓ_2 passes through the point $(4, -2)$, its parametric form is:

$(x, y) = (4, -2) + k(\Delta x, \Delta y),$

where the direction vector $(\Delta x, \Delta y)$ satisfies

$\frac{\Delta y}{\Delta x} = m_2 = -\frac{3}{2}.$

Choosing $\Delta x = 1, \Delta y = -\frac{3}{2}$, we get

$(x, y) = \left(4 + k, -2 - \frac{3}{2}k\right).$

Step 4: Check options for the parametric points:

- Option 1: $(4 + 3k, -2 + 2k)$ – slope $\frac{2}{3}$, not perpendicular.
- Option 2: $(4 - 2k, -2 - 3k)$ – slope $\frac{3}{2}$, not negative reciprocal.
- Option 3: $\boxed{(4 + k, -2 - \frac{3}{2}k)}$ – slope $-\frac{3}{2}$, correct.
- Option 4: $(4 + 6k, -2 + \frac{3}{2}k)$ – slope $\frac{1}{4}$, incorrect.

Thus, the correct answer is $\boxed{(4 + k, -2 - \frac{3}{2}k)}$

79) Choice B is the correct answer.

Step 1: Start with the original equation

$5x + 2y = 8$

Step 2: Apply the translation

We are translating:

- 4 units to the left → replace x with $x + 4$
- 6 units down → replace y with $y + 6$

Substitute into the equation:

$5(x + 4) + 2(y + 6) = 8$

Step 3: Expand the expression

$5x + 20 + 2y + 12 = 8$

$5x + 2y + 32 = 8$

Step 4: Simplify

$5x + 2y = 8 - 32 = -24$

So, the new equation of the line is:

$5x + 2y = -24$

Step 5: Find the new y-intercept

Set $x = 0$:

$5(0) + 2y = -24 \Rightarrow 2y = -24 \Rightarrow y = -12$

Thus, the correct answer is $\boxed{-12}$.

80) Choice A is the correct answer.

Here's a step-by-step breakdown of how to derive the function representing the total final cost $C(x, w)$:

Step 1: Setup cost:

The consultancy charges a one-time setup cost of $\$1,150.75$, which is added to the total cost.

$\text{Setup cost} = 1,150.75$

Step 2: Weekday cost:

The weekday rate is $\$328.40$ per day for weekdays, and there are $x - w$ weekday days (total days minus weekend days).

$\text{Weekday cost} = 328.40 \times (x - w)$

Step 3: Weekend cost:

The weekend premium is $\$419.65$ per day for weekends, and there are w weekend days.

$\text{Weekend cost} = 419.65 \times w$

Step 4: Discount on weekdays (if applicable):

If $x > 20$, there is a 5% discount on the total weekday cost.

This means the weekday cost is multiplied by 0.95 (which is $1 - 0.05$).

$\text{Discounted weekday cost} = 0.95 \times 328.40 \times (x - w) \quad \text{if} \quad x > 20$

Step 5: Subtotal (before tax):

The subtotal is the sum of the setup cost, the (possibly discounted) weekday cost, and the weekend cost. If $x > 20$, the weekday cost is discounted; otherwise, the full weekday rate is used.

$$\text{Subtotal} = \begin{cases} 1150.75 + 0.95 \times 328.40 \times (x - w) + 419.65w, & \text{if } x > 20 \\ 1150.75 + 328.40 \times (x - w) + 419.65w, & \text{otherwise} \end{cases}$$

Step 6: Tax:

A fixed 8.75% tax is applied to the entire subtotal after any discount.

This is calculated by multiplying the subtotal by 1.0875 (which is $1 + 0.0875$).

$\text{Final cost} = 1.0875 \times \text{Subtotal}$

Step 7: Final formula:

Combining all the pieces, the final cost function becomes:

$$C(x,w)=\begin{cases}1.0875\times[1150.75+0.95\times328.40\times(x-w)+419.65w], & \text{if } x>20\\ 1.0875\times[1150.75+328.40\times(x-w)+419.65w], & \text{otherwise}\end{cases}$$

This correctly reflects all aspects of the pricing: the setup cost, weekday rate, weekend premium, discount on weekdays for $x>20$, and the 8.75 tax.

Thus, option A is the correct answer.

81) Choice B is the correct answer.

We are given the function $f(x)=\frac{7}{3}x+b$ and the x-intercept $(13,0)$.

We are tasked with finding the y-intercept of the function $f(-2x+5)+4$.

Step 1: Determine $b$using the x-intercept

The x-intercept of a function is the point where $y=0$.

For the function $f(x)=\frac{7}{3}x+b$, we know the x-intercept occurs at $x=13$, so we set $f(13)=0$:

$f(13)=\frac{7}{3}(13)+b=0$

Simplifying this equation:

$\frac{7}{3}\times13+b=0$

$\frac{91}{3}+b=0$

$b=-\frac{91}{3}$

Thus, the function is $f(x)=\frac{7}{3}x-\frac{91}{3}$.

Step 2: Substitute into the modified function $f(-2x+5)+4$

We now substitute the expression for $f(x)$ into $f(-2x+5)$:

$f(-2x+5)=\frac{7}{3}(-2x+5)-\frac{91}{3}$

Distribute $\frac{7}{3}$:

$f(-2x+5)=\frac{7}{3}(-2x)+\frac{7}{3}(5)-\frac{91}{3}$

$f(-2x+5)=-\frac{14}{3}x+\frac{35}{3}-\frac{91}{3}$

$f(-2x+5)=-\frac{14}{3}x-\frac{56}{3}$

Now, add 4 to this result:

$f(-2x+5)+4=-\frac{14}{3}x-\frac{56}{3}+4$

Convert 4 into a fraction with denominator 3:

$4=\frac{12}{3}$

Thus:

$f(-2x+5)+4=-\frac{14}{3}x-\frac{56}{3}+\frac{12}{3}$

$f(-2x+5)+4=-\frac{14}{3}x-\frac{44}{3}$

Step 3: Find the y-intercept of $f(-2x+5)+4$

The y-intercept occurs when $x=0$.

Substituting $x=0$into the expression for $f(-2x+5)+4$:

$f(0)=-\frac{14}{3}(0)-\frac{44}{3}=-\frac{44}{3}$

Thus, the y-intercept is $\boxed{-\frac{44}{3}}$.

82) Choice D is the correct answer.

Given:

$5-3\left|\frac{4}{5}+\frac{2}{3}x\right|=\frac{11}{4}+2k$

To find the values of k such that the equation has no solution.

Step 1: Simplify the equation

Start by isolating the absolute value expression.

Subtract 5 from both sides:

$-3\left|\frac{4}{5}+\frac{2}{3}x\right|=\frac{11}{4}+2k-5$

Simplify the right-hand side:

$\frac{11}{4} - 5 = \frac{11}{4} - \frac{20}{4} = \frac{-9}{4}$

Now the equation becomes:

$-3\left|\frac{4}{5} + \frac{2}{3}x\right| = \frac{-9}{4} + 2k$

Step 2: Solve for the absolute value term

Next, divide both sides by -3 (note that dividing by a negative number reverses the inequality):

$\left|\frac{4}{5} + \frac{2}{3}x\right| = \frac{3}{4} - \frac{2}{3}k$

Step 3: Analyze the conditions for no solution

For an absolute value equation to have no solution, the right-hand side of the equation (the expression inside the absolute value) must be negative, because the absolute value of a number can never be negative.

Therefore, we require:

$\frac{3}{4} - \frac{2}{3}k < 0$

Step 4: Solve the inequality for k

We now solve the inequality:

$\frac{3}{4} - \frac{2}{3}k < 0$

First, subtract $\frac{3}{4}$ from both sides:

$-\frac{2}{3}k < -\frac{3}{4}$

Multiply both sides by -1(and reverse the inequality):

$\frac{2}{3}k > \frac{3}{4}$

Next, multiply both sides by $\frac{3}{2}$ to solve for k:

$k > \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$

Step 5: Conclusion

The inequality $k > \frac{9}{8}$ gives the values of k such that the equation has no solution.

Thus, the correct inequality is: $\boxed{k > \frac{9}{8}}$

83) Choice C is the correct answer.

We are given two equations for two paths:

- $4x - 7y = 10$
- $ax + by = c$

We are also given that these two paths are perpendicular.

Our goal is to find which of the following systems also represents two perpendicular paths.

Step 1: Find the slope of the first line

The first equation is:

$4x - 7y = 10$

We need to rewrite this equation in slope-intercept form $y = mx + b$, where m is the slope.

Solve for y:

$4x - 7y = 10 \quad \Rightarrow \quad -7y = -4x + 10 \quad \Rightarrow \quad y = \frac{4}{7}x - \frac{10}{7}$

Thus, the slope of the first line is:

$m_1 = \frac{4}{7}$

Step 2: Find the slope of the second line

The second equation is:

$ax + by = c$

Rewrite it in slope-intercept form:

$by = -ax + c \quad \Rightarrow \quad y = -\frac{a}{b}x + \frac{c}{b}$

Thus, the slope of the second line is:

$m_2 = -\frac{a}{b}$

Step 3: Apply the condition for perpendicular lines

Two lines are perpendicular if the product of their slopes is -1.

So, for the two lines to be perpendicular, the condition is:

$$m_1 \cdot m_2 = -1$$

Substitute the slopes we found:

$$\frac{4}{7} \cdot \left(-\frac{a}{b}\right) = -1$$

Simplify:

$$-\frac{4a}{7b} = -1 \quad \Rightarrow \quad \frac{4a}{7b} = 1$$

Thus:

$$\frac{a}{b} = \frac{7}{4}$$

Step 4: Check each option to find the system with perpendicular lines

Now that we know that for the lines to be perpendicular, $\frac{a}{b} = \frac{7}{4}$, let's check the options:

Option A:

Equations:

1. $8x - 14y = 20$

2. $3ax + 2by = 9$

First equation:

$$8x - 14y = 20 \quad \Rightarrow \quad y = \frac{8}{14}x - \frac{20}{14} = \frac{4}{7}x - \frac{10}{7}$$

Slope of the first line:

$$m_1 = \frac{4}{7}$$

Second equation:

$$3ax + 2by = 9 \quad \Rightarrow \quad y = -\frac{3a}{2b}x + \frac{9}{2b}$$

Slope of the second line:

$$m_2 = -\frac{3a}{2b}$$

For perpendicular lines, we need:

$$\frac{4}{7} \cdot \left(-\frac{3a}{2b}\right) = -1$$

Simplify:

$$-\frac{12a}{14b} = -1 \quad \Rightarrow \quad \frac{12a}{14b} = 1 \quad \Rightarrow \quad \frac{a}{b} = \frac{7}{6}$$

This does not match the required condition $\frac{a}{b} = \frac{7}{4}$, so Option A is not correct.

Option B:

Equations:

1. $x - 10.5y = 12$

2. $2ax - 3by = 6$

First equation:

$$x - 10.5y = 12 \quad \Rightarrow \quad y = \frac{1}{10.5}x - \frac{12}{10.5}$$

Slope of the first line:

$$m_1 = \frac{1}{10.5}$$

Second equation:

$$2ax - 3by = 6 \quad \Rightarrow \quad y = \frac{2ax}{3b} - \frac{6}{3b}$$

Slope of the second line:

$$m_2 = \frac{2a}{3b}$$

For perpendicular lines, we need:

$$\frac{1}{10.5} \cdot \frac{2a}{3b} = -1$$

This does not satisfy the condition for perpendicularity, so Option B is not correct.

Option C:

Equations:

1. $16x - 28y = 40$

2. $7x + 4y = 8$

First equation:

$16x - 28y = 40 \quad \Rightarrow \quad y = \frac{16}{28}x - \frac{40}{28} = \frac{4}{7}x - \frac{10}{7}$

Slope of the first line:

$m_1 = \frac{4}{7}$

Second equation:

$7x + 4y = 8 \quad \Rightarrow \quad y = -\frac{7}{4}x + 2$

Slope of the second line:

$m_2 = -\frac{7}{4}$

For perpendicular lines, we need:

$\frac{4}{7} \cdot \left(-\frac{7}{4}\right) = -1$

This satisfies the condition for perpendicularity, so Option C is correct.

Option D:

Equations:

1. $9x - 7y = 13$

2. $ax - 3by = 2$

First equation:

$9x - 7y = 13 \quad \Rightarrow \quad y = \frac{9}{7}x - \frac{13}{7}$

Slope of the first line:

$m_1 = \frac{9}{7}$

Second equation:

$ax - 3by = 2 \quad \Rightarrow \quad y = \frac{a}{3b}x - \frac{2}{3b}$

Slope of the second line:

$m_2 = \frac{a}{3b}$

For perpendicular lines, we need:

$\frac{9}{7} \cdot \frac{a}{3b} = -1$

This does not satisfy the condition for perpendicularity, so Option D is not correct.

Step 5: Conclusion

The correct system that represents two perpendicular paths is: $\boxed{16x - 28y = 40 \quad \text{and} \quad 7x + 4y = 8}$

84) Choice A is the correct answer.

We are given the system of equations:

$4.5x - 2.1y = 63$

$9x - 4.2y = 126$

We are tasked with finding which ordered pair (x, y) represents a solution to both equations for any real number r.

Step 1: Solve the system of equations

We begin by simplifying the equations to see if there is a pattern or relationship between x and y.

First, observe that the second equation is simply a multiple of the first equation.

To make this clearer, multiply the first equation by 2:

$2 \times (4.5x - 2.1y = 63) \implies 9x - 4.2y = 126$

This is exactly the second equation, so both equations are equivalent.

This means they represent the same line, and any solution to one equation will be a solution to the other.

Step 2: Express $y$in terms of x

We can now solve for y in terms of x from the first equation:

$4.5x - 2.1y = 63$

$-2.1y = 63 - 4.5x$

$y = \frac{4.5x-63}{2.1}$

Step 3: Generalize the solution

For any real number r, we can choose $x = r$, and substitute this value into the equation for y:

$y = \frac{4.5r-63}{2.1}$

Thus, the ordered pair (x, y) is $\left(r, \frac{4.5r-63}{2.1}\right)$.

Thus, the correct option is: $\boxed{\left(r, \frac{4.5r-63}{2.1}\right)}$

85) Choice C is the correct answer.

Given:

Area of an equilateral triangle $= \frac{625\sqrt{3}}{4}$.

Step 1: Recall the formula for the area of an equilateral triangle

$\text{Area} = \frac{\sqrt{3}}{4}s^2,$

where s is the side length.

Step 2: Set the area equal and solve for s^2

$\frac{\sqrt{3}}{4}s^2 = \frac{625\sqrt{3}}{4}.$

Multiply both sides by 4:

$\sqrt{3}s^2 = 625\sqrt{3}.$

Divide both sides by $\sqrt{3}$:

$s^2 = 625.$

Step 3: Solve for s

$s = \sqrt{625} = 25.$

Step 4: Find the perimeter

$\text{Perimeter} = 3s = 3 \times 25 = 75.$

Thus, the correct answer is $\boxed{75}$

86) Choice C is the correct answer.

We are given:

Height of an equilateral triangle:

$h = \left(\frac{3k+2}{5}\right)\sqrt{3}$

Area of the triangle:

$A = \frac{676\sqrt{3}}{25}$

Let's denote the side length of the equilateral triangle as s.

The height h of an equilateral triangle in terms of side s is:

$h = \frac{\sqrt{3}}{2}s$

Step 1: Equating expressions for height

$\left(\frac{3k+2}{5}\right)\sqrt{3} = \frac{\sqrt{3}}{2}s$

Divide both sides by $\sqrt{3}$:

$\frac{3k+2}{5} = \frac{s}{2}$

$\Rightarrow s = \frac{2(3k+2)}{5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

Step 2: Use the area formula of an equilateral triangle

$\text{Area} = \frac{\sqrt{3}}{4}s^2$

Given:

$$\frac{\sqrt{3}}{4}s^2 = \frac{676\sqrt{3}}{25}$$

Cancel $\sqrt{3}$ from both sides:

$$\frac{1}{4}s^2 = \frac{676}{25}$$

$$\Rightarrow s^2 = \frac{4 \cdot 676}{25} = \frac{2704}{25}$$

Step 3: Solve for k

Recall from (1):

$$s = \frac{2(3k+2)}{5}$$

$$\Rightarrow s^2 = \left(\frac{2(3k+2)}{5}\right)^2 = \frac{4(3k+2)^2}{25}$$

So:

$$\frac{4(3k+2)^2}{25} = \frac{2704}{25}$$

$$\Rightarrow 4(3k+2)^2 = 2704$$

$$\Rightarrow (3k+2)^2 = \tfrac{2704}{4} = 676$$

$$\Rightarrow 3k+2 = \sqrt{676} = 26$$

$$\Rightarrow 3k = 24$$

$$\Rightarrow k = 8$$

Thus, the correct answer is $\boxed{8}$

87) Choice D is correct.

Given:

- Center of the circle: $(-6.25,\ 5.5)$
- Radius: $r = \sqrt{110}$

The general form of circle equation:

$$x^2 + y^2 + ax + by + c = 0$$

Step 1: Write the standard form of the circle equation

$$(x-h)^2 + (y-k)^2 = r^2$$

Substitute:

$$(x+6.25)^2 + (y-5.5)^2 = 110$$

Step 2: Expand each square

$$(x+6.25)^2 = x^2 + 2 \times 6.25 \times x + 6.25^2 = x^2 + 12.5x + 39.0625$$

$$(y-5.5)^2 = y^2 - 2 \times 5.5 \times y + 5.5^2 = y^2 - 11y + 30.25$$

Step 3: Substitute expanded terms

$$x^2 + 12.5x + 39.0625 + y^2 - 11y + 30.25 = 110$$

Combine constants:

$$x^2 + y^2 + 12.5x - 11y + 69.3125 = 110$$

Step 4: Bring all terms to one side for the general form

$$x^2 + y^2 + 12.5x - 11y + 69.3125 - 110 = 0$$

$$x^2 + y^2 + 12.5x - 11y - 40.6875 = 0$$

Step 5: Identify c

$$c = -40.6875$$

Step 6: Round c to the nearest integer

$$c \approx \boxed{-41}$$

88) Choice C is the correct answer.

We are given:

- A circle centered at $(5, -2)$
- A point (b, c) on the circle where the tangent line has slope $\frac{4}{3}$
- (b, c) lies on the line $c = \frac{1}{2}b - 5$

Step 1: Use Perpendicular Slope

The radius from the center $(5, -2)$ to the point of tangency (b, c) is perpendicular to the tangent line.

The slope of the radius is therefore the negative reciprocal of the tangent's slope:

$$\text{Slope of radius} = -\frac{1}{\left(\frac{4}{3}\right)} = -\frac{3}{4}$$

So the slope between the points $(5, -2)$ and (b, c) is $-\frac{3}{4}$:

$$\frac{c-(-2)}{b-5} = -\frac{3}{4}$$

$$\Rightarrow \frac{c+2}{b-5} = -\frac{3}{4}$$

Step 2: Solve the equation

Cross-multiplying:

$$4(c + 2) = -3(b - 5)$$

Distribute:

$$4c + 8 = -3b + 15$$

$$4c + 3b = 7 \quad \text{(Equation 1)}$$

Step 3: Use the line equation

We're told the point (b, c) lies on the line $c = \frac{1}{2}b - 5$.

Substitute into Equation 1:

$$4\left(\frac{1}{2}b - 5\right) + 3b = 7$$

$$\Rightarrow 2b - 20 + 3b = 7$$

$$\Rightarrow 5b = 27$$

$$\Rightarrow b = \frac{27}{5}$$

Thus, the correct answer is $\boxed{\frac{27}{5}}$

89) Choice B is the correct answer.

We are given a triangle with sides of lengths x, $x + 2$, and $2x$, and each side serves as the diameter of a semicircle drawn externally.

We are to find the total area of the three semicircles in the form $\omega\pi$, given that $x = \frac{6}{5}$.

Step 1: Side lengths

Given: $x = \frac{6}{5}$

Sides of the triangle:

- First side $= x = \frac{6}{5}$
- Second side $= x + 2 = \frac{6}{5} + 2 = \frac{16}{5}$
- Third side $= 2x = 2 \cdot \frac{6}{5} = \frac{12}{5}$

Step 2: Semicircle area formula

The area of a semicircle with diameter d is:

$$A = \frac{1}{2} \cdot \pi \cdot \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8}$$

We will use this for all three semicircles.

Step 3: Calculate each area

Let's compute the area of each semicircle:

For diameter $x = \frac{6}{5}$:

$$A_1 = \frac{\pi}{8} \cdot \left(\frac{6}{5}\right)^2 = \frac{\pi}{8} \cdot \frac{36}{25} = \frac{36\pi}{200} = \frac{9\pi}{50}$$

For diameter $x + 2 = \frac{16}{5}$:

$$A_2 = \frac{\pi}{8} \cdot \left(\frac{16}{5}\right)^2 = \frac{\pi}{8} \cdot \frac{256}{25} = \frac{256\pi}{200} = \frac{64\pi}{50}$$

For diameter $2x = \frac{12}{5}$:

$$A_3 = \frac{\pi}{8} \cdot \left(\frac{12}{5}\right)^2 = \frac{\pi}{8} \cdot \frac{144}{25} = \frac{144\pi}{200} = \frac{36\pi}{50}$$

Step 4: Total area

$A_{\text{total}} = A_1 + A_2 + A_3 = \frac{9\pi}{50} + \frac{64\pi}{50} + \frac{36\pi}{50} = \frac{109\pi}{50}$

So, the total area is $w\pi = \frac{109\pi}{50}$, hence:

$w = \frac{109}{50}$

Thus, the correct answer is $\boxed{w = \frac{109}{50}}$

90) Choice D is the correct answer.

We are given:

- $\triangle PQR$ is isosceles with $PQ = PR$
- Point S lies on side QR, with $QS = \frac{9}{13} \cdot QR$
- Points T and U are on PQ and PR, respectively, such that:

  $\angle TQS = \angle URS$

  $ST = 18$

We are to find SU

Step 1: Use of equal angles

We are told that:

$\angle QTS = \angle RUS$

Also as $PQ = PR$

$\implies \angle TQS = \angle URS$

This implies that triangles $\triangle TQS$ and $\triangle URS$ are similar by AA similarity,

So, we get:

$\triangle TQS \sim \triangle URS$

From similarity:

$\frac{TQ}{UR} = \frac{QS}{SR}$ and $\frac{ST}{SU} = \frac{QS}{SR}$

Step 2: Use the given ratio on side QR

We're told:

$QS = \frac{9}{13} \cdot QR \Rightarrow SR = QR - QS = \left(1 - \frac{9}{13}\right) QR = \frac{4}{13} QR$

So,

$\frac{QS}{SR} = \frac{\frac{9}{13} QR}{\frac{4}{13} QR} = \frac{9}{4}$

Step 3: Use similarity to find SU

Since:

$\frac{ST}{SU} = \frac{QS}{SR} = \frac{9}{4}$

And we are told:

$ST = 18$

We get:

$\frac{18}{SU} = \frac{9}{4} \Rightarrow SU = \frac{18 \cdot 4}{9} = 8$

Thus, the correct answer is $\boxed{8}$

91) Choice D is the correct answer.

Let's follow the steps below:

Step 1: Understand the geometric configuration

Since $\overline{PQ} \parallel \overline{RS}$, we know that triangles $\triangle PQT$ and $\triangle RST$ are similar by AA similarity.

This is because corresponding angles formed by parallel lines are congruent.

Step 2: Set up the proportion

The corresponding sides of the similar triangles will have the same ratio.

Therefore, we can set up the following proportion:

$$\frac{QT}{TS} = \frac{PT}{TR}$$

Step 3: Substitute the known values

We are given the lengths:

- $QT = 5.6$
- $TS = 8.4$
- $PT = 10.5$

Substituting into the proportion:

$$\frac{5.6}{8.4} = \frac{10.5}{TR}$$

Step 4: Simplify the left-hand side

Simplifying $\frac{5.6}{8.4}$ gives:

$$\frac{5.6}{8.4} = \frac{2}{3}$$

So, the proportion becomes:

$$\frac{2}{3} = \frac{10.5}{TR}$$

Step 5: Solve for TR

Cross-multiply to solve for TR:

$$2 \times TR = 3 \times 10.5$$

$$2 \times TR = 31.5$$

$$TR = \frac{31.5}{2} = 15.75$$

Step 6: Calculate the total length of PR

The total length of PR is the sum of PT and TR:

$$PR = PT + TR = 10.5 + 15.75 = 26.25$$

The correct length of PR is $\boxed{26.25}$.

92) Choice D is the correct answer.

Given:

One acute angle: $4.5x - 37.5^\circ$

One obtuse angle: $9.5x - 52.5^\circ$

The total measure of one acute angle and two obtuse angles is: $-14x + w^\circ$

Step 1: Add up one acute angle and two obtuse angles:

$$< /strong > \text{Total} = (4.5x - 37.5) + 2(9.5x - 52.5)$$

$$= 4.5x - 37.5 + 19x - 105$$

$$= (4.5x + 19x) - (37.5 + 105) = 23.5x - 142.5$$

Step 2: Equate with the given expression:

$$23.5x - 142.5 = -14x + w$$

Step 3: Solve for w:

Add $14x$ to both sides:

$$23.5x + 14x - 142.5 = w$$

$$37.5x - 142.5 = w$$

Thus, the correct answer is $\boxed{w = 37.5x - 142.5}$

93) Choice C is the correct answer.

Given:

- Right triangle XYZ with right angle at Z.
- $XZ = 52$ units.
- Area of $\triangle XYZ = 416$ square units.
- $MN \parallel XY$, with M on XZ, N on YZ.
- $MN = 13$ units.
- $XY > YZ$.
- Find length of YN.

Step 1: Coordinates

Place Z at origin:

$$Z = (0, 0), \quad X = (52, 0), \quad Y = (0, h).$$

Step 2: Find h from area

$$\frac{1}{2} \times 52 \times h = 416 \implies 26h = 416 \implies h = 16.$$

So,

$$Y = (0, 16).$$

Step 3: Calculate XY

$$XY = \sqrt{(52 - 0)^2 + (0 - 16)^2} = \sqrt{2704 + 256} = \sqrt{2960} \approx 54.4 > 16.$$

Step 4: Parametrize points M and N

$$M = (m, 0), \quad N = (0, n),$$

with $0 \le m \le 52, 0 \le n \le 16.$

Step 5: Slope of XY

$$m_{XY} = \frac{16-0}{0-52} = -\frac{4}{13}.$$

Step 6: Slope of MN

$$m_{MN} = \frac{n-0}{0-m} = -\frac{n}{m}.$$

Since $MN \parallel XY$:

$$-\frac{n}{m} = -\frac{4}{13} \implies \frac{n}{m} = \frac{4}{13} \implies n = \frac{4}{13}m.$$

Step 7: Length of $MN = 13$

$$MN = \sqrt{m^2 + n^2} = 13.$$

Substitute $n = \frac{4}{13}m$:

$$\sqrt{m^2 + \left(\frac{4}{13}m\right)^2} = 13,$$

$$\sqrt{m^2 + \frac{16}{169}m^2} = 13,$$

$$m\sqrt{1 + \frac{16}{169}} = 13,$$

$$m\sqrt{\frac{185}{169}} = 13,$$

$$m = \frac{13}{\sqrt{\frac{185}{169}}} = 13 \times \sqrt{\frac{169}{185}} = \frac{169}{\sqrt{185}}.$$

Step 8: Calculate n

$$n = \frac{4}{13}m = \frac{4}{13} \times \frac{169}{\sqrt{185}} = \frac{52}{\sqrt{185}}.$$

Step 9: Approximate numerical value

$$\sqrt{185} \approx 13.601,$$

$$n \approx \frac{52}{13.601} \approx 3.82.$$

So:

$$\overline{YN} = 16 - 3.82 = 12.18$$

$$\boxed{\overline{YN} \approx 12.2 \text{ units}}$$

Thus, the correct answer is $\boxed{12.2 \text{ units.}}$

94) Choice D is the correct answer.

Step 1: Understand the given information

Region	Number of Animals	Mean Weight (kg)	Total Weight (kg)
A	x	24.5	$24.5x$
B	$2x + 10$	31.2	$31.2(2x + 10)$
C	$0.5x$	Unknown	$0.5x \cdot w_C$

Let the total number of animals $= x + (2x + 10) + 0.5x = 3.5x + 10$

Let the total weight $= 24.5x + 31.2(2x + 10) + 0.5x \cdot w_C$

Step 2: Use the formula for average

The overall average weight is given to be 29.5, so:

$$\frac{24.5x + 31.2(2x + 10) + 0.5x \cdot w_C}{3.5x + 10} = 29.5$$

Step 3: Multiply both sides by $(3.5x + 10)$ to eliminate the denominator:

$$24.5x + 31.2(2x + 10) + 0.5x \cdot w_C = 29.5(3.5x + 10)$$

Step 4: Solve for w_C

Move the known terms to the right:

$$0.5x \cdot w_C = 29.5(3.5x + 10) - 24.5x - 31.2(2x + 10)$$

Now divide both sides by $0.5x$:

$$w_C = \frac{29.5(3.5x + 10) - 24.5x - 31.2(2x + 10)}{0.5x}$$

Thus, the correct answer is $\boxed{\frac{29.5(3.5x + 10) - 24.5x - 31.2(2x + 10)}{0.5x}}$

95) Choice C is the correct answer.

Let's follow the below steps:

Step 1: Calculate the Mean:

The mean (average) of the set is:

$\text{Mean} = \frac{15+8+20+y+y}{5} = \frac{43+2y}{5}$

Step 2: We now set the median equal to the mean in all cases

Case 1: $y \leq 8$

The set becomes $\{y, y, 8, 15, 20, \}$.

The median is the middle value, which is 8.

The mean is:

$\frac{43+2y}{5} = 8$

Multiply both sides by 5:

$43 + 2y = 40$

Subtract 43 from both sides:

$2y = -3$

Divide by 2:

$y = -\frac{3}{2}$

Since $y = -1.5$ satisfies $y \leq 8$, this case provides a valid solution.

Case 2: $y \geq 20$

The set becomes $\{8, 15, 20, y, y\}$. Arranged in increasing order, we get:

$\{8, 15, 20, y, y\}$

The median is again the middle value, which is 20.

The mean is:

$\frac{43+2y}{5} = 20$

Multiply both sides by 5:

$43 + 2y = 100$

Subtract 43 from both sides:

$2y = 57$

$y = \frac{57}{2} = 28.5$

Since $y = 28.5$ satisfies $y \geq 20$, this case provides a valid solution.

Case 3: $8 < y \leq 15$

Ordered list: $\{8, y, y, 15, 20\}$

Median = y

Set mean = median:

$\frac{43+2y}{5} = y => 43 + 2y = 5y$

$43 = 3y => y = \frac{43}{3} \approx 14.3$

Check if $y = 14.3$ falls in $8 < y \leq 15$: Yes (valid solution).

Case 4: $15 < y < 20$

Ordered list: $\{8, 15, y, y, 20\}$

Median = y

Set mean = median:

$\frac{43+2y}{5} = y \implies y = \frac{43}{3} \approx 14.3$

But $y = 14.3$ does not satisfy $15 < y < 20$.

No solution here.

Step 4: Sum all possible values of y

The only valid solutions are $y = 28.5$, $y = -1.5$ and $y = 14.3$

Therefore, the sum of all possible values of y is $28.5 + (-1.5) + 14.3 = 41.3$

Thus, the correct answer is 41.3.

96) Choice C is the correct answer.

Let's follow the below steps:

Step 1: Number of students who attended the first session

We are given that:

900 students registered for the first session.

So,

$\text{Students in Session } 1 = 900$

Step 2: Number of students who attended both the first and second sessions

We are told that:

60% of those who attended the first session also attended the second session.

So,

$\text{Students in Sessions } 1 \text{ and } 2 = 60\% \text{ of } 900 = \frac{60}{100} \times 900 = 540$

Step 3: Number of students who attended all three sessions

We are also told that:

25% of the students who attended both the first and second sessions also attended the third session.

So,

$\text{Students in Sessions } 1, 2, \text{ and } 3 = 25\% \text{ of } 540 = \frac{25}{100} \times 540 = 135$

Thus, $\boxed{135}$ students attended all three training sessions.

97) Choice C is the correct answer.

Step 1: Understand the Original Data Set A

- Number of values: 37
- Minimum value: 230
- Maximum value: 278
- Mean: 244
- Standard deviation: 9

Step 2: Understand What Happens in Set B

- Create set B by removing only two values: 230 (min) and 278 (max)
- New number of values: 35

Step 3: Analyze the Mean of Set B

Let's estimate how the mean is affected.

The mean of a data set is:

$$\bar{x} = \frac{\text{sum of all values}}{n}$$

So the sum of set A is:

$244 \times 37 = 9028$

Now subtract the two removed values:

$9028 - 230 - 278 = 8520$

Number of values in set B: 35

$$\text{New mean} = \frac{8520}{35} \approx 243.43$$

Conclusion: The mean decreased slightly from 244 to approximately 243.43

Step 4: Analyze the Standard Deviation of Set B

Standard deviation reflects the spread of values.

- Set A: Min $= 230$, Max $= 278$ → these are extreme values.
- Removing both extremes reduces the range and variability.

Standard deviation will definitely decrease because the dataset becomes more tightly clustered without those outliers.

Conclusion: Standard deviation in set B is less than in set A.

Step 5: Compare to Each Option

Option	Mean	Std. Dev	Verdict
1. Both greater	×	×	Incorrect
2. Std. dev greater, mean same	×	×	Incorrect
3. Std. dev less, mean slightly less	✓	✓	**Correct**
4. Both equal	×	×	Incorrect

Thus, the standard deviation of data set B is less than that of data set A, and the mean is slightly less than that of data set A.

98) Choice C is the correct answer.

Step 1: Calculate the percent decrease from 2022 to 2023

Number of subscribers in 2022: $712,500$

Number of subscribers in 2023: $683,000$

Decrease in subscribers:

$712,500 - 683,000 = 29,500$

Percent decrease:

$\frac{29{,}500}{712{,}500} \times 100 = \frac{29{,}500}{712{,}500} \times 100 \approx 4.14\%$

Step 2: Calculate the predicted percent decrease from 2023 to 2024

The analyst predicted this decrease is one-third of the previous decrease:

$\frac{1}{3} \times 4.14\% \approx 1.38\%$

Step 3: Calculate expected subscribers at the end of 2024

Decrease from 2023 to 2024:

$1.38\% \text{ of } 683,000 = 0.0138 \times 683,000 \approx 9,425.4$

Expected subscribers at end of 2024:

$683,000 - 9,425.4 \approx 673,574.6$

Rounding to the nearest whole number: $673,575$

Thus, the analyst expected about $\boxed{673,575}$ subscribers at the end of 2024.

99) Choice C is the correct answer.

Step 1: Find the actual wingspan using the first model's scale

- First model's scale: 1 cm → 12 m actual.
- The model wingspan is 9.5 cm.
- So actual wingspan $= 9.5 \times 12 = 114$ meters.

Step 2: Find the wingspan on the second model using its scale

- Second model's scale: 1 cm → 24 m actual.
- The same actual wingspan (114 meters) in the second model gives:

$$\text{Second model wingspan} = \frac{114}{24} = 4.75 \text{ cm}.$$

Step 3: Compare the wingspans of the two models

- First model wingspan: 9.5 cm
- Second model wingspan: 4.75 cm
- Clearly, 4.75 cm is half of 9.5 cm: $4.75 = \frac{9.5}{2}$.

Final Answer:

The wingspan of the second model aircraft will be $\frac{1}{2}$ as long as the wingspan of the first model aircraft.

100) Choice C is the correct answer.

Step 1: Understand the given information

- Regular rate: $\$22.50$ per hour
- Regular hours: first 36 hours
- Overtime rate: 1.5 times the regular rate

$$\Rightarrow \text{Overtime rate} = 1.5 \times 22.50 = 33.75 \text{ dollars/hour}$$

- Bonus: $\$45$ if more than 40 hours worked
- Total earnings for the week: $\$1,260$

We are to find the total number of whole hours worked that week.

Step 2: Let the total hours worked be x

Let x be the total number of hours worked.

There are three cases:

1. $x \leq 36$: All hours at regular rate

2. $36 < x \leq 40$: First 36 hours at regular rate, remaining at overtime rate (no bonus)

3. $x > 40$: First 36 hours at regular rate, next $(x - 36)$ hours at overtime rate, and a $\$45$ bonus

Since $x > 40$ gives a bonus and total earnings are relatively high ($\$1,260$), we'll assume $x > 40$ and verify.

Step 3: Write the total earnings equation for $x > 40$

$$\text{Total earnings} = (\text{Regular pay}) + (\text{Overtime pay}) + \text{Bonus}$$

$$\Rightarrow 1260 = (36 \times 22.50) + (x - 36) \times 33.75 + 45$$

Step 4: Calculate regular pay

$$36 \times 22.50 = 810$$

Substitute:

$$1260 = 810 + (x - 36) \times 33.75 + 45$$

Step 5: Combine known values

$$810 + 45 = 855$$

$$\Rightarrow 1260 = 855 + (x - 36) \times 33.75$$

Step 6: Solve for x

Subtract 855 from both sides:

$$1260 - 855 = (x - 36) \times 33.75$$

$$405 = (x - 36) \times 33.75$$

Divide both sides by 33.75:

$$x - 36 = \frac{405}{33.75} = 12$$

$$\Rightarrow x = 36 + 12 = 48$$

Thus, the correct answer is $\boxed{48}$ hours